PROCEEDINGS

OF THE

NATIONAL ACADEMY OF SCIENCES

Volume 2 FEBRUARY 15, 1916 Number 2

PERSONAL EQUATION AND STEADINESS OF JUDGMENT IN THE ESTIMATION OF THE NUMBER OF OBJECTS IN MODERATELY LARGE SAMPLES

By J. Arthur Harris

STATION FOR EXPERIMENTAL EVOLUTION, COLD SPRING HARBOR, N. Y.

Received by the Academy, December 20, 1915

Investigations in experimental evolution, involving as they necessarily do the recording of the characteristics of large numbers of individuals, afford the opportunity of obtaining quite incidentally extensive series of data upon errors of observation or estimation. The present paper states the major results of one such series of records obtained at the Station for Experimental Evolution during the course of investigation requiring the counting of large numbers of beans for mass weighings, germination tests, etc.

The chief advantages of such observations lie in the facts that they are carried out under quite natural working conditions, with none of the artificiality of the laboratory test, and that they represent far larger experiments than the average professional psychologist is able to make. Comprising as they do 28 experiments due to three observers all of whom carried on the work at considerably separated intervals over a period of two years, during which they made over 15,000 estimates with determined errors, the constants have a reliability which cannot possibly be attributed to short series.

The routine of this work was so organized that it consisted in part of a series of attempts to lay out samples of a definite number (25, 50, 100, or 200) which was constant for considerable periods. The error of each estimate was at once determined and recorded.

Two characteristics of the series of errors of estimation made by the three observers are here considered—personal equation and steadiness of judgment.

By personal equation we understand a definite bias in a given direction. In a series of estimates by an observer the errors in excess of the true value may be no more numerous and no greater in amount than those in defect. The average of the deviation of the estimates from the true number of objects will then be 0 plus or minus a small amount due to the errors of random sampling. Such an observer may be said to have no personal equation. Other individuals, however, may have a definite tendency to err on one side of verity in their evaluations. Such may be said to have a positive or a negative personal equation, as the case may be. Personal equation is measured by the mean, regarding signs, of the deviation of the samples from their ideal value.

But quite without reference to personal equation, one observer may be more erratic than another, estimating now far too high, now far too low. By steadiness of judgment we mean consistency in estimation as measured by the closeness with which the errors of estimation cluster around their mean value. Steadiness of judgment may be expressed in the absolute terms of the standard deviation of the errors of estimation about their mean (S.D.), or in the relative terms of the coefficient of variation (C.V.).

$$S.D. = \sqrt{\frac{\text{Sum of (Deviations from Mean)}^2}{\text{Total estimates}}}$$

which here is most easily calculated from the formula

$$(S.D.)^2 = \Sigma(d^2)/N - [\Sigma(d)/N]^2,$$

where Σ is the conventional summation sign, N is the number of estimates and d indicates the deviation of the estimate from the true number of objects, *i. e.*, the actual number laid out less the required number; and

$$C.V. = 100\, S.D./M,$$

where M is the constant number which the observer seeks to lay out plus or minus the observed personal equation, as the sign of the latter may indicate.

In much of the work in which personal equation is a factor the observer is not able to check his estimates against the true values, and so attempt at each successive observation to profit by his previous experience. In these experiments each observer made a persistent effort to improve. This was based on a knowledge of the immediately preceding errors, and consisted in a constant effort to lay out exactly the desired number of seeds. Thus the influence of experience upon both

personal equation and steadiness of judgment may be determined from these data.

The problems here taken under consideration fall, therefore, into two groups. *First*, those having to do with the existence of personal equation in the estimation of the number of objects in samples and of differences in personal equation and steadiness of judgment from individual to individual. *Second*, the influence of previous experience upon personal equation and steadiness of judgment.

In the case of all three observers there is a slight but significant personal equation, which, notwithstanding the constant effort to improve, persisted throughout the two years during which the experiments were intermittently made. *In only three out of the twenty-eight experiments did the observer lay out samples of too small average size.* In a large number of the individual experiments the personal equation is certainly statistically significant (trustworthy) in comparison with its probable error.

From the experimental data taken as a whole one cannot conclude that there is any demonstrated difference between the personal equation of the three observers, although the figures do suggest that the bias of observer *D* may be slightly greater than that of either of the others. All have a bias in the direction of laying out more than the intended number of seeds, but that one is worse than another cannot be asserted.

In a high proportion of the individual experiments the differences between the personal equations of the three observers are statistically significant in relation to their probable errors. This is true in cases in which (for example) *B* has a greater personal equation than *C*, as well as in these in which she has a smaller personal equation.

The probable explanation of this result seems to be that the observers vary somewhat in their personal equation from experiment to experiment, just as they vary from time to time in general health, physiological tone, and mental vigor, alertness, or whatever one may care to call it. As a result of this variation from time to time one observer may show an abnormally high personal equation in a particular experiment in which a second observer shows an unusually low one. On another occasion the condition may be exactly reversed.

Thus *in an individual experiment* one observer may seem to be decidedly better than another. *In the long run* there is no *fully demonstrated* difference between them.

For steadiness of judgment there is no absolute standard comparable with the zero mean deviation of the personal equation. The data show a coefficient of variation of about 6.9% in the case of Observer *B* and *C*,

and of 8.7% in the case of Observer *D*, who has a decidedly greater scatter in her estimates—that is a far less steady judgment—than either of the other observers. Indeed, in every individual experiment her standard deviation is higher than that of either of the two other experimenters.

Thus while there is no certain differentiation among the experimenters in personal equation, they differ distinctly in steadiness of judgment.

The influence of previous experience upon personal equation or steadiness of judgment may be most succinctly expressed in terms of the correlation between some quantitative measure of the amount of previous experience and the measures of personal equation and steadiness of judgment.

In these experiments the errors of observation were recorded in sequence. A group of fifty consecutive estimates with the accompanying determinations of the errors constituted a 'period.' In determining correlations one must deal with a number of subgroups for each period. It is most convenient to divide each half daily period of 50 estimates into five consecutive 'trials,' each of 10 estimates. For each of these 'trials' the mean personal equation and the standard deviation of the errors must be computed. Thus in obtaining the constants discussed here it was first necessary to compute 1520 means and 1520 standard deviations, which were then treated as units in computing the correlations.

The main problems involved in the question of experience are two: Is there a change in personal bias as a result of constant effort to improve and opportunity for improvement? Does the judgment become steadier, *i.e.*, does the observer make less erratic estimates, as a result of experience?

Both of these questions are really twofold. Is there an improvement from period to period? Is there an improvement within the period? In short, does the worker improve both from estimate to estimate in the same half daily period and also from period to period?

Personal equation seems to be remarkably little influenced by experience. In some experiments it increases, in others it decreases. The correlations may be either positive or negative in sign. Numerically they are generally low, and are in great part insignificant in comparison with their probable errors. Taken as a whole the results indicate a slight reduction in personal equation as a result of experience from period to period. Within the period there is no demonstrable influence of experience upon personal equation.

Steadiness of judgment is in rather conspicuous contrast with per-

sonal equation, in that it is unmistakably influenced by previous experience. The correlations between the number of previous trials within the period and steadiness of judgment and between the number of previous periods of experience and steadiness of judgment are numerically low, but almost without exception indicate that as experience becomes greater the scatter of the individual estimates about their mean value becomes less. Probably the rate of this change is not uniform, but is most rapid at first and then falls off.

The full data and discussion are appearing in two papers in the *Psychological Review*.

POLYPEPTIDE-HYDANTOINS

By Treat B. Johnson

SHEFFIELD SCIENTIFIC SCHOOL, YALE UNIVERSITY

Received by the Academy, December 24, 1915

Carbon dioxide is one of the products of decomposition when certain proteins undergo hydrolysis, under normal conditions, by digestion with aqueous solutions of acids and alkalis. Mörner[1] observed the formation of this acid anhydride during an investigation of the action of hydrochloric acid (sp. gr. 1.124) on horn at 92°, but no quantitative determination of the gas was made and no special significance attributed by him to its formation. Lippich[2] confirmed this observation several years later and showed that this anhydride is a normal product of hydrolysis of other proteins. He also made the important observation that the quantity actually formed is dependent on the nature of the hydrolytic agent employed. Quantitative determinations of the amount of the gas evolved from several proteins under specific conditions revealed the interesting fact that the maximum quantity is obtained when an alkali, as potassium or barium hydroxide, is used as the hydrolytic agent. In no case did Lippich fail to detect the presence of this substance among his products of hydrolysis. The actual percentages obtained by hydrolysis of five different proteins with potassium hydroxide solution are recorded in Table I.

For his acid hydrolyses Lippich used 33% sulphuric acid. When these same proteins were broken down by heating with this reagent entirely different analytical results were obtained. The combinations in the proteins productive of carbon dioxide were more resistant to hydrolytic changes, under these conditions, and the maximum amount of this gas was not obtained until after 25–27 hours digestion. The percentages found are recorded in Table I.

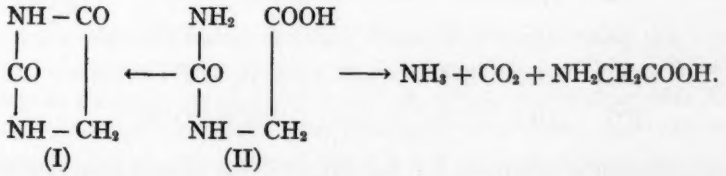
TABLE I

PROTEIN	CARBON DIOXIDE: By acid hydrolysis per cent	CARBON DIOXIDE: By alkali hydrolysis per cent
Parahämoglobin	0.352 0.332 0.370	2.09
Egg albumin	0.336	2.77
Serum globulin	0.364	2.41 2.38
Elastin	0.130	0.680
Keratin	0.372	4.63 4.53

In the course of investigations now in progress in this laboratory, dealing with the study of new hydantoin and thiohydantoin combinations of biochemical interest, I had occasion to determine whether gaseous products are evolved by hydrolysis of pure fibroin and also casein. I now find that both these proteins break down on hydrolysis with sulphuric acid with evolution of carbon dioxide and obtained from casein 0.35% of its weight in the form of this gas, while three quantitative experiments with fibroin gave the values 0.20, 0.27, and 0.25% respectively. When boiling 30% barium hydroxide solution was used for the hydrolysis of fibroin (the action of barium hydroxide on casein is being investigated), we obtained the values 1.08 and 1.05% of carbon dioxide. In other words, in the cases of the two proteins elastin and fibroin about five times as much carbon dioxide is produced by alkaline hydrolysis as is obtained by digestion with sulphuric acid. By inspection of Table I it will be seen that the greatest increase was obtained by hydrolysis of keratin. The results obtained with fibroin have special significance because the production of carbon dioxide in this case does not result from the breaking down of cystin, as this protein is free from sulphur.

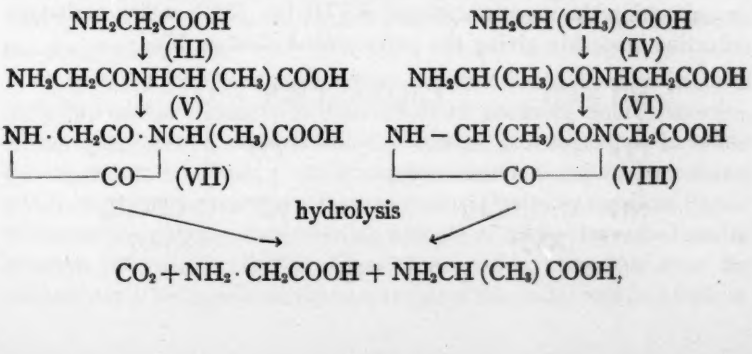
The marked difference in behavior towards acids and alkalis and the further fact that the quantity of carbon dioxide formed by hydrolysis is not proportional to the amount of arginine present in the respective proteins, suggest that urea combinations are the precursors of this gaseous hydrolytic product. Carbon dioxide would be a normal product of hydrolysis of such groupings and their destruction would be more easily accomplished by the action of alkalis than with acids. Lippich has advanced the idea that uramido acids, of which hydantoic acid II is the prototype, are the types of urea combinations which functionate in these changes. Such groupings, as is well known, are hydrolyzed by alkalis with formation of α-amino acids, carbon dioxide and ammonia.

On the other hand, when heated with acids, they easily undergo inner condensation with transformation into their corresponding anhydrides or *hydantoins* I. The latter compounds are very resistant to further hydrolysis with acids as has been shown by investigations in this laboratory.



While this explanation of Lippich's is in concordance with the chemical nature of hydantoic acids, on the other hand these compounds (II) do not represent the only types of uramido combinations which can undergo hydrolysis with formation of carbon dioxide and α-amino acids. One point needs to be taken into consideration here and that is the fact, that for every molecule of carbon dioxide formed by hydrolysis of a hydantoic acid II an equivalent amount of ammonia must also be obtained. Apparently this ratio CO_2:NH_3 has never been established quantitatively in the case of a single protein

Theoretically it is possible to link together two α-amino acids in a cyclic urea combination in such a manner that the resulting compound will undergo hydrolysis with production only of carbon dioxide and α-amino acids. If we choose glycocoll III and alanine IV as the two amino acids and combine them in the form of hydantoins as represented by formulas VII and VIII, two isomeric cyclic combinations will be obtained which will fulfil the above conditions. In other words, they are cyclic derivatives of the two isomeric dipeptides—glycylalanine V and alanylglycine VI respectively, in which the characteristic polypeptide groupings of the dipeptides have been preserved. Such combinations are the representatives of a new class of hydantoins,



to which we have assigned the name—*polypeptide-hydantoins*. The hydantoin derivatives of the polypeptides—glycylphenylalanine IX, phenylalanylglycine X, glycyltyrosine XI and tyrosineglycine XII are now being investigated. Their formulas are:

(IX) $NH_2CH_2CO \cdot NHCH(CH_2C_6H_5)COOH$

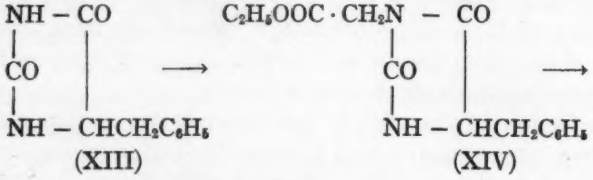
(X) $NH_2 \cdot CH(CH_2C_6H_5) \cdot CONH \cdot CH_2COOH$

(XI) $NH_2 \cdot CH_2CONH \cdot CH(CH_2C_6H_4 \cdot OH)COOH$

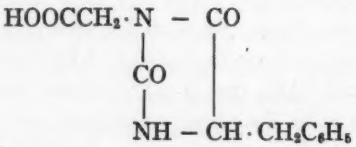
(XII) $NH_2CH(CH_2C_6H_4OH)CONHCH_2COOH$.

The polypeptide-hydantoin XV (m. 184–185°) has already been synthesized and two methods have been developed for its preparation which are apparently of general application:

(1) Alkylation of 4-benzylhydantoin[3] XIII with ethyl chloracetate with formation of the ester XIV (m. 157°), which is easily converted into the *polypeptide-hydantoin* XV by saponification.

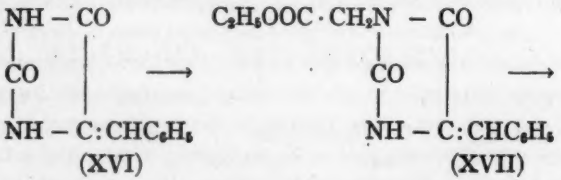


(XIII) (XIV)

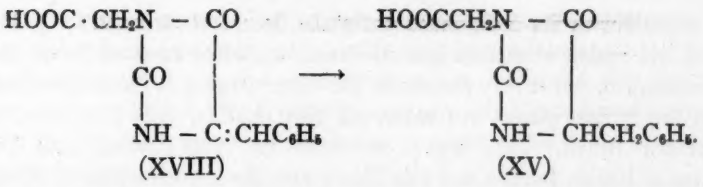


(XV)

(2) Alkylation of 4-benzalhydantoin[4] XVI, with ethyl chloracetate with formation of the ester XVII (m. 174°). The latter gives by saponification the unsaturated acid XVIII (m. 258°), which undergoes reduction smoothly giving the *polypeptide-hydantoin* XV.



(XVI) (XVII)



(XVIII) (XV)

Other methods for synthesizing hydantoin compounds of this type (and their sulphur analogues) are being developed. This work will include not only the study of hydantoin derivatives of dipeptides but also tripeptide combinations of analogous constitution, and also an investigation of their behavior towards hydrolytic agents and enzymes.

[1] Mörner, *Zs. physiol. Chem.*, **34**, 207 (1901).

[2] Lippich, *Ibid.*, **90**, 441 (1914).

[3] Wheeler and Hoffman, *Amer. Chem. J.*, **45**, 368 (1911).

[4] Ruhemann and Cunnington, *London, J. Chem. Soc.*, **75**, 958 (1899); Ruhemann and Stapleton, *Ibid.*, **77**, 246 (1900); Wheeler and Hoffman, *loc. cit.*

RECENT EXPLORATIONS IN THE CACTUS DESERTS OF SOUTH AMERICA

By J. N. Rose

DEPARTMENT OF BOTANICAL RESEARCH, CARNEGIE INSTITUTION OF WASHINGTON

Read before the Academy, November 16, 1915. Received January 6, 1916

When the cactus investigation for the Carnegie Institution of Washington was taken up a few years ago with Dr. N. L. Britton, a definite plan for field work in the deserts of North and South America was outlined. And since the Cactus family is confined to America, this meant a survey of its entire distribution.

The exploration of the deserts of South America was referred to me, and I have spent two seasons in exploring these regions; the first on the west coast, in Peru, Bolivia, and Chile, and the second on the east coast, in Brazil and Argentina. The exploration was confined chiefly to the deserts, as the cacti as a class are desert loving plants. A cactus desert, however, does not necessarily mean a desert like the Sahara or the desert of Arizona.

We found as a result of our investigation that South America contains six great cactus regions: (1) The desert of northern South America, including northern Venezuela and Colombia. This region we have not yet studied at first hand. (2) The great Peruvian and Chilean desert which extends from northern Peru to central Chile and from the Pacific Ocean to the top of the Andes, having a length of 2000 miles and a breadth of 50 to 300 miles. (3) The desert of Argentina, extending from the central part of Patagonia along the east side of the Andes well into Bolivia.

It resembles in its component parts the desert of Arizona. (4) The semi-arid region of eastern central Brazil, including most of Bahia and Pernambuco. It is very similar to the desert region of Santo Domingo and the typical genera are nearly all West Indian. (5) The desert of southern Brazil. This region we have not yet studied. (6) The states of Rio de Janeiro and São Paulo and the southern part of Minas Geraes, Brazil.

The last region is one of abundant rainfall and where all ordinary cacti would be killed. Here, however, the cacti not only grow on rocky knobs and along the beaches, but especially on the trunks of trees. Under the last named condition these plants find the same zerophytic conditions that their relatives find which grow in New Mexico and western Texas. They attach their roots to the bark of trees, their stems are reduced to long, shoe-string-like bodies, while the spines are reduced to hairs or they disappear altogether. About 40 of these epiphytic species, mostly belonging to the genus Rhipsalis, have developed in this region and they represent a most interesting group.

We have made large collections in South America in the fields visited; and we have ascertained that many species of cacti had never before been collected, and that many of those which had been collected had been poorly described and often wrongly classified.

ON THE ALBEDO OF THE PLANETS AND THEIR SATELLITES

By Henry Norris Russell

PRINCETON UNIVERSITY OBSERVATORY

Read before the Academy, November 17, 1915. Received January 10, 1916

1. The most suitable definition of albedo for astronomical purposes appears to be that proposed by Bond[1] in 1861, namely, the ratio of the whole amount of light reflected in all directions from a sphere illuminated by parallel rays to the amount of light incident on the sphere.

2. The albedo A of any planet, according to this definition, is the product of two factors, one of which depends only on the size of the planet, its distances from the earth and sun, and its brightness at the full phase, while the other depends upon the way in which the brightness varies at different phases. The first factor, which may be called p, can be calculated from known data for all the planets. Its value depends mainly upon the material of the surface, being high if this is nearly white, and low if it is dark colored. The second factor, q, can be computed only when the planet is observable over a considerable range of phase, so that the law of variation of its brightness with phase can be determined, and its values are known only for the moon and the planets

of the terrestrial group. For Jupiter and the remoter planets, however, estimates can be made which are not likely to be much more than 15% in error. This factor depends mainly upon the *texture* of the planet's surface, being high if this is smooth, and low if it is rough and covered with irregularities whose shadows darken considerable areas at phases remote from the full.

3. The relative brightness of the sun and stars, as seen from the earth, has been determined with surprising accuracy,—the results of several observers, by radically different methods, being in excellent agreement. The sun's stellar magnitude, on the Harvard scale, according to the mean of the observations of Zöllner, Ch. Fabry, Ceraski and W. H. Pickering, is -26.72 ± 0.04,—which is equivalent to saying that the sun appears to be 123 thousand million times as bright as a standard first magnitude star. The photographic magnitude of the sun, according to King and Birck, is -25.93, and its color index $+0.79$, agreeing very closely with the average for stars of similar spectrum (Class G).

4. The law of variation of the moon's brightness with phase is very well determined by the observations of J. Herschel, Bond, Zöllner, W. H. Pickering, King, Stebbins and Brown, and Wislicenus,—the results of all seven agreeing satisfactorily with a mean curve. The full moon is 8.7 times brighter than the first quarter, and 10.0 times brighter than the last quarter. The remarkable falling off in brightness between the full and half moon shows that the lunar surface must be very rough, as was first pointed out by Zöllner. The difference between the waxing and waning moon arises from the greater extent of the dark *maria* on the eastern half of the visible disk (as Stebbins has shown).

5. The results of different observers for the brightness of the mean full moon, compared with the sun or stars, are discordant. J. Herschel's observations[2] have been reduced anew, and an error which had crept into the earlier reduction corrected. The weighted mean of those determinations which are not obviously affected by systematic error makes the visual magnitude -12.55, and the ratio of sunlight to mean full moonlight 465,000, —with an uncertainty of fully 10%. The photographic magnitude, -11.37, has been well determined by King. It shows that moonlight is redder than sunlight,—in agreement with the spectro-photometric measures of Wilsing and Scheiner.

6. Müller's data for the major planets[3] have been adopted, with a correction of -0.06 mag. to reduce to the Harvard scale, except for Uranus and Neptune; Pickering's magnitudes for the asteroids,[4] and Guthnick's for the satellites of Jupiter and Saturn.[5] The color indices of Venus, Mars, Jupiter and Saturn have been derived from comparison of Müller's visual and King's photographic observations.[6]

7. Very's observations of the intensity of the earthshine[7] indicate that the mean full earth, as seen from the moon, appears 40 times as bright as the mean full moon, seen from the earth, and that the stellar magnitude of the earth, as seen from the sun, would be about −3.5, with an uncertainty of at least 25%, or 0.20 mag.

8. The intensity of sunlight from the zenith, according to H. H. Kimball, is 103,000 metre-candles. That of mean full moonlight, according to several observers, is 0.24 metre-candle. A standard candle, if of approximately the same color as the stars, would appear as bright as a star of the first magnitude if placed at a distance of 1.09 kilometres.

9. Table 1 gives the values finally derived for the albedo of the various planets and satellites, and related quantities.

TABLE I

Object	Mag. at Mean Opp.	Mag. at Unit Distance	Semi-diameter	p	q	Albedo A	Color-Index	Photographic Albedo
Moon	−12.55	+0.40	2".40	0.105	0.694	0.073	+1.18	0.051
Mercury	− 2.94	−0.88	3.45	0.164	0.42	0.069	.....	.....
	− 2.12	−0.06		0.077	0.72	0.055	.....	.....
Venus	− 4.77	−4.06	8.55	0.492	1.20	0.59	+0.78	0.60
Mars	− 1.85	−1.36	4.67	0.139	1.11	0.154	+1.38	0.090
Jupiter	− 2.29	−8.99	95.23	0.375	1.5:	0.56:	+0.50	0.73:
Saturn	+ 0.89	−8.67	77.95	0.420	1.5:	0.63:	+1.12	0.47:
Uranus	+ 5.74	−6.98	36.0	0.42	1.5:	0.63:	.....	.....
Neptune	+ 7.65	−7.00	34.5	0.49	1.5:	0.73:	.....	.....
Ceres	+ 7.15	+3.70	0.53	0.10	0.55:	0.06:	.....	.....
Pallas	+ 7.84	+4.38	0.34	0.13	0.55:	0.07:	.....	.....
Juno	+ 8.95	+5.74	0.14	0.22	0.55:	0.12:	.....	.....
Vesta	+ 6.04	+3.50	0.27	0.48	0.55:	0.26:	.....	.....
Jupiter								
Satellite I	+ 5.54	−1.16	2.38	0.46	1.5:	0.69:	.....	.....
" II	+ 5.69	−1.01	2.08	0.51	1.5:	0.76:	.....	.....
" III	+ 5.08	−1.62	3.62	0.30	1.5:	0.45:	.....	.....
" IV	+ 6.26	−0.44	3.49	0.11	1.5:	0.16:	.....	.....
Titan	+ 8.30	−1.26	2.9	0.33	1.5:	0.50:	.....	.....
The Earth								
Lommel-Seeliger law		−3.46	8.79	0.27	1.64	0.45	.....	.....
Lambert's law		−3.52	"	0.29	1.50	0.43	+0.45?	0.6?
Observed law for Venus		−3.80	"	0.37	1.20	0.45	.....	.....
" " " Moon		−4.40	"	0.65	0.70	0.45	.....	.....

Column 2 gives the stellar magnitude at mean opposition (in the case of Mercury and Venus, at full phase, unit distance from the earth, and mean distance from the sun); column 3 the magnitude for full phase and unit distance from the sun and the observer; column 4 the adopted mean angular semidiameter at unit distance; the next three columns, the values of p, q, and A, defined as above; and the last two, the color-index and the photographic albedo (A) in the cases where these are

known. Two sets of values are given for Mercury, corresponding to the two empirical formulae for the variation of its brightness with phase given by Müller, and four for the earth, representing the results derived from Very's observations on the assumption that the variation of the brightness of the earth with phase follows four different laws, derived from theory or from observation of other bodies.

10. These values of the albedo of the various bodies are in entire agreement with the current views of their constitution. For Venus and the outer planets, which are generally supposed to be covered with clouds, the albedo is very near the value found by Abbot for terrestrial clouds (0.65). For Mars, Mercury and the moon the albedo is comparable with that of ordinary rocks, as it is also for three of the four asteroids. Even the high value for Vesta can be matched by some whitish terrestrial rocks; but the still higher values for the inner satellites of Jupiter are rather remarkable.

11. The value here found for the earth's albedo is intermediate between those of the cloudy and cloudless planets, and agrees very closely with Abbot's estimate[8] of 0.37, based on the known cloudiness of the earth's atmosphere. It is only half as great as that which Very has derived from the same observations, but the discrepancy is easily explicable. The discussions of the observations by Very and by the writer agree in showing that the albedo of the earth (more precisely, the value of the constant called p above) is a little more than five times as great as that of the moon (if Zöllner's value for the brightness of the latter is adopted). But the value of the moon's albedo used by Very (0.174) is what Zöllner[9] calls the "true albedo"—which is the value obtained after a large and very uncertain correction for the assumed influence of the irregularities of the surface, according to a theory which has later been found to contain a serious error.[10] Zöllner's observations themselves lead to the value 0.080 for p, and when this correction is made, the discrepancy disappears.

The full paper, with much more extensive references, will be published in the *Astrophysical Journal*.

[1] *Proc. Amer. Acad. Arts Sci.*, 8, 232 (1861).

[2] *Results of Astronomical Observations made at the Cape of Good Hope* (London, 1847), pp. 353–374.

[3] *Potsdam Pub. Astrophysik.*, Bd. 8, Tl. IV (1893).

[4] *Harvard Coll. Obs. Cir., No. 169* (1911).

[5] *Astr. Nachr.*, 198, 251 (1914).

[6] *Ann. Obs. Harvard Coll.*, 59, 261–264.

[7] *Astr. Nachr.*, 196, 269–290 (1912).

[8] *Smithsonian Inst., Ann. Astrophys. Obs.*, 2, 161–163.

[9] *Photometrische Untersuchungen* (Leipzig, 1865), p. 162.

[10] See Müller, *Photometrie der Gestirne* (Leipzig, 1897), p. 77.

QUANTUM RELATIONS IN PHOTO-ELECTRIC PHENOMENA

By R. A. Millikan

RYERSON PHYSICAL LABORATORY, UNIVERSITY OF CHICAGO

Received by the Academy, December 21, 1915

For the past ten years I have been engaged with experiments which were designed for the sake of subjecting Einstein's photo-electric quantum-theory equation to searching experimental tests, and although I have at times thought that I had evidence which was irreconcilable with that equation, the longer I have worked and the more completely I have eliminated sources of error the better has the equation been found to predict the observed results. I shall present herewith the barest sketch of six consequences of that equation and their experimental verification. Preliminary reports on some of these results have already been made[1] and detailed reports will be found in forthcoming numbers of the *Physical Review*.

Einstein's equation[2] grew out of a semi-corpuscular quantum theory of radiation. The assumption was that light consists of bundles or 'quanta' of electromagnetic energy which shoot out explosively from the emitting body and travel through space as localized units until they are suddenly absorbed by the atoms of matter upon which they fall. The energy in each light-unit was assumed equal to $h\nu$, in which h is Planck's 'wirkungs-quantum' and ν is the frequency of the oscillator which emits the light. Upon absorption this energy was assumed to be transformed into the kinetic energy of an escaping negative electron whose energy of escape from a metal illuminated by light of frequency ν was thus given by $\frac{1}{2}mv^2 = h\nu - p$, in which p was the work necessary to separate the electron from the surface of the metal. The maximum energy of escape is measured by $(V_0 + K)e$ in which e is the electronic charge, K the contract EMF between the emitting plate and the opposed Faraday cylinder which catches the electron, and V_0 the potential difference which must be externally applied just to stop the photo-current to this cylinder. The assertions contained in the equation $\frac{1}{2}mv^2 = h\nu - p$ are that:

1. There is a definite maximum energy of electronic emission under the stimulation of a given frequency ν. (This has recently been denied by Ramsauer.[3])

2. There is a linear relation between V_0 and ν.

3. The slope of the $V_0\nu$ line multiplied by e is exactly Planck's 'h.'

4. The intercept of this $V_0\nu$ line on the ν axis is the frequency ν_0 at which the illuminated substance first becomes photo-sensitive.

5. The contact EMF between the illuminated plate and the Faraday cylinder is given by,

$$\text{Contact EMF} = \frac{h\nu_0 - h\nu'_0}{e} - (V_0 - V'_0),$$

in which ν_0 and ν'_0 are the frequencies at which the cylinder and the

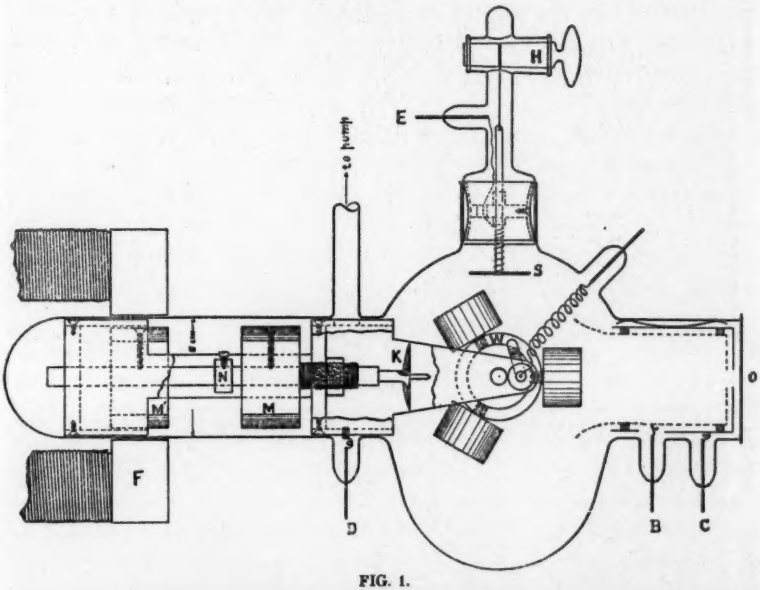


FIG. 1.

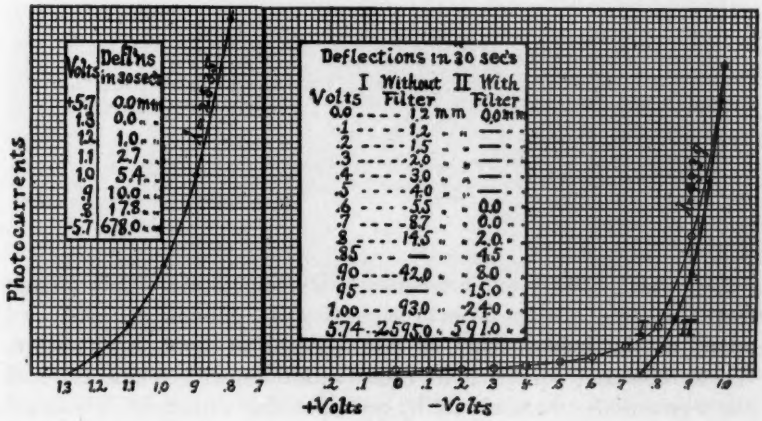


FIG. 2.

plate respectively first become photo-sensitive, and V_0 and V'_0 are the respective maximum potentials necessary to stop discharge into the cylinder from the plate and from another plate made from the substance of the cylinder.

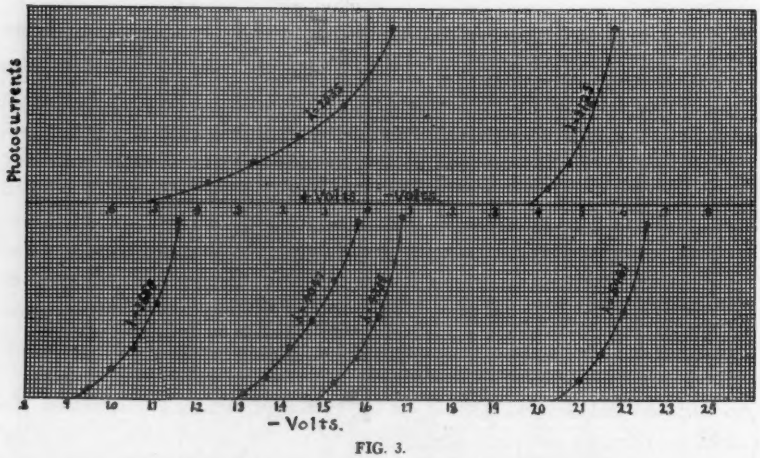


FIG. 3.

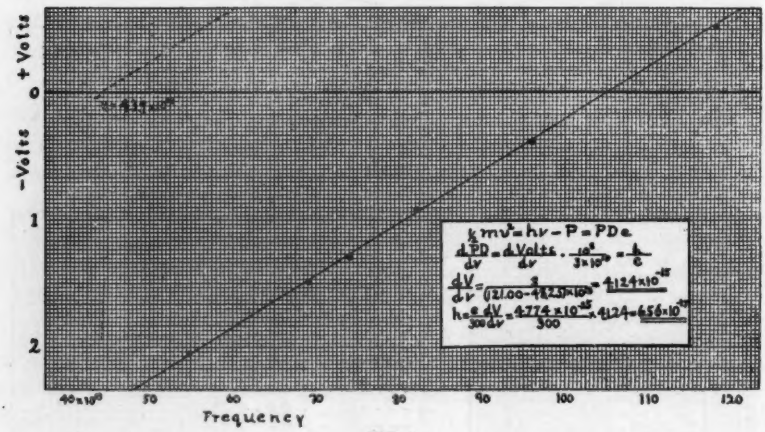


FIG. 4.

6. Contact EMF's are independent of temperature. This follows from Einstein's equation taken in connection with our now well confirmed proof[4] of the independence of photo-potentials upon temperature. Where, however, surface films cause variations with temperature of photo-potentials, there should be corresponding variations in contact EMF.

The experiments are made with a device shown in figure 1 by which clean new surfaces of potassium, sodium, and lithium can be produced by shaving in an extreme vacuum, and photo currents and contact EMF's

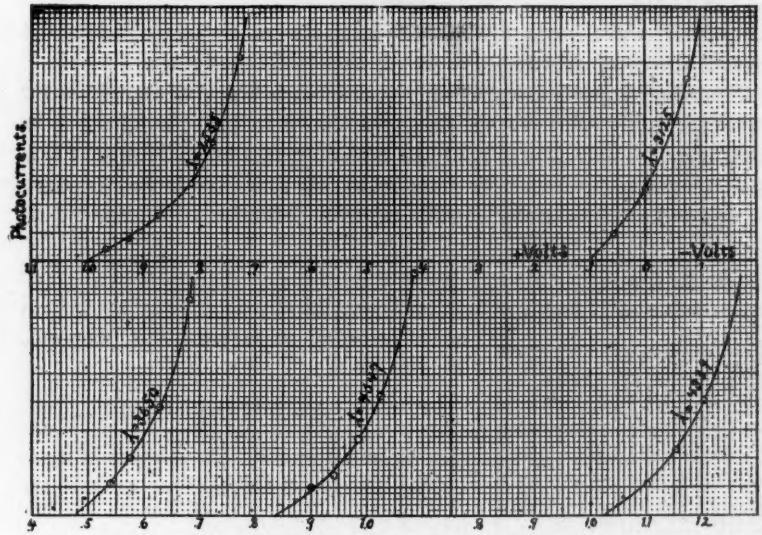


FIG. 5.

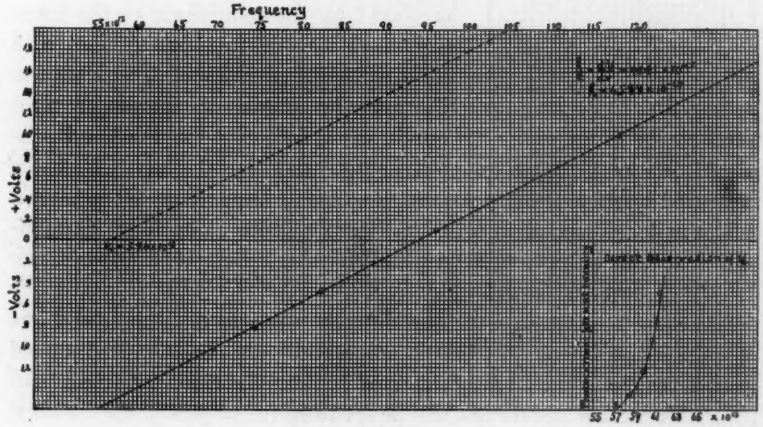


FIG. 6.

measured immediately thereafter. The tube has a projection toward the reader not shown in the diagram with the aid of which the wheel w may be rotated by means of an electromagnet similar to F and a third armature similar to M and M'.

The data on lithium shown in figure 2 seems to establish assertion 1, the difference between curves I and II taken with the mercury line 4339 and a Hilgar monochromator showing how the true shape (II) of the photo-current-potential curve was entirely falsified by a little stray short wave-length light (see I) until a filter of aesculin which cut out all waves of shorter wave-length then 4339 was used.

Assertions 2 and 3 are strikingly verified in figures 3 and 4, the latter of which is the plot of the intercepts shown in figure 3, against the frequency. These figures relate to measurements on sodium, in which the saturation currents were from 75 to 500 times the largest currents in figure 3. These latter correspond to a deflection of 80 mm. in 30 seconds.

Similar measurements on lithium are shown in figures 5 and 6. The mean value of Planck's h thus photo-electrically determined should not be in error by more than 0.5 per cent. This value is

$$h = 6.57 \times 10^{-27}$$

(see figures 4 and 6). The value of e involved in this determination of h is the author's value[5] 4.774×10^{-10}.

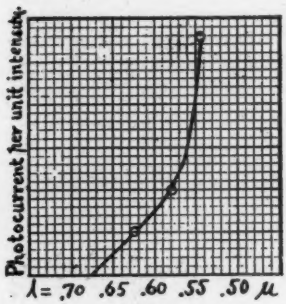


FIG. 7.

The correctness of assertion 4 is shown in figure 6 by the coincidence of the intercept $\nu_0 = 57.0 \times 10^{13}$ (see dotted line) with the direct determination of ν_0 with saturation currents shown on the lower right hand corner of figure 6. To obtain the ν_0 intercept the $V_0\nu$ line is displaced in the direction of positive potentials by the amount of the measured contact EMF. Also in the case of the sodium the ν_0 shown in figure 4 corresponds to the wave length 0.68μ. The direct determination of this long wave length limit for the sodium is shown in figure 7. The agreement is perfect.

Assertion 5 was tested for two different surfaces of lithium and one of sodium and the contact EMF computed by the equation in 5 agreed in each case with the directly observed contact EMF to within less than 2%. The details of this test will be found in the papers in the *Physical Review*.

Assertion 6 has not been tested in this work but Schottky[6] has recently measured the contact EMF's between white hot wires and cold cylinders and found results which agree, within the fairly wide limits of

uncertainty, with the values which hold between the same metals at ordinary temperatures.

So far then as experiment has thus far gone, Einstein's equation seems to be an exact statement of the energies of emission of corpuscles under the influence of light waves.

Nevertheless the physical theory which gave rise to it seems to me to be wholly untenable. Be this as it may, however, the photo-electric results herewith presented constitute the best evidence thus far found for the correctness of the fundamental assumption of quantum theory, namely, the discontinuous or explosive emission of energy by electronic oscillators. They furnish the most direct and most tangible evidence which we yet have for the actual physical reality of Planck's h.

[1] R. A. Millikan, *Physic. Rev.*. Ser. 2, 4, 73 (1914); *Ibid.*, 6, 55 (1915).
[2] Einstein, *Ann. Physik.*, Ser. 4, 17, 132 (1905) and 20, 199 (1906).
[3] Ramsauer, *Ann. Physik.*, 45, 1120 (1914), also 45, 961.
[4] Millikan and Winchester, *Physic. Rev.*, 24, 16 (1906), and *Phil. Mag.*, Ser. 6, 14, 188 (1907).
[5] Millikan, *Physic. Rev.* 2, 143 (1913).
[6] Schottky, *Ann. Physik.*, 44, 1011 (1914).

THE CHEMICAL ACTIVITY OF THE IONS OF HYDROCHLORIC ACID DETERMINED BY ELECTROMOTIVE FORCE MEASUREMENTS

By James H. Ellis

RESEARCH LABORATORY OF PHYSICAL CHEMISTRY,
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Received by the Academy, January 22, 1916

The fact that the laws of perfect solutions which are conformed to by unionized or slightly ionized substances in dilute aqueous solutions are subject to large deviations in the case of largely ionized substances (salts, strong acids and bases) even at small concentrations makes it necessary, in the absence of any theoretical explanation of the deviations, to treat dilute solutions of these substances like concentrated solutions of other substances, namely, to determine experimentally the behavior of the separate substances, with the hope that this empirical study may then lead to generalizations. Now the most important characteristic of ionizing substances is the chemical activity which results from their ionization, or more specifically the (mass-action) effect which their ions exercise in determining chemical equilibria. This effect in the case of theoretically perfect solutes is proportional to the concentration of the ions; but in the case of deviating solutes there must be substituted

for it a new quantity, which may be regarded as the 'effective ion-concentration' and which has been appropriately called by Lewis[1] the activity of the ions. This quantity has been shown by Lewis to be thermodynamically related to various other properties of solutions, thereby on the one hand increasing its practical significance and on the other affording independent means of evaluating it.

The most general of these thermodynamic relations, one indeed which may well be regarded as the best practical definition of activity, is that afforded by the equation $F_1 - F_2 = RT \log (a_1/a_2)$ in which R is the perfect-gas constant and $F_1 - F_2$ represents the decrease in the free-energy of the system attending the transfer at the absolute temperature T of one mol of any substance (thus of an ion) from a solution of any concentration in which its activity is a_1 to another solution of any concentration in which its activity is a_2; the free-energy decrease being defined in general to be equal to the maximum work W producible by the change in the state of the system under consideration diminished by the attendant increase in the product of its volume and pressure (that is, $F_1 - F_2 = W - (p_2v_2 - p_1v_1)$).

The most direct way of determining the free-energy-decrease attending the transfer of ions from one concentration to another, and thereby of determining their relative activities, is the measurement of the electromotive force of cells in which such a transfer takes place; and it is with such a study of the ions of hydrochloric acid that this investigation deals. Namely, measurements have been made of the electromotive force at 18, 25, and 35° of cells of the form H_2 (1 atm.), HCl (at various concentrations), Hg_2Cl_2 (solid) + Hg. If two such cells are considered to be placed in series in opposition to each other, the changes at the electrodes of the two cells compensate each other, and the net change in state when one faraday (F coulombs) of electricity passes through it is the transfer of 1 HCl or of 1 H^+ and 1 Cl^- from one solution to the other. Considering the ions, we have therefore the relation:

$$(E_2 - E_1)\,\mathrm{F} = F_1 - F_2 = 2\,\mathrm{RT} \log (a_1/a_2),$$

in which $E_2 - E_1$ is the difference in the electromotive force of two cells in which the acid has the free-energies F_2 and F_1 and its ions have the activities a_2 and a_1, respectively. For a_2 and a_1 we may substitute α_2c_2 and α_1c_1 in which α_2 and α_1 are activity-coefficients (analogous to ionization-coefficients) representing the factors by which the concentrations c_2 and c_1 of the acid must be multiplied to give the activities of the ions.

The mean corrected values of the observed electromotive force in

volts and the values of the free-energy-decrease in joules calculated from them by the equation $F_1 - F_2 = 96500\,E \times 4.182$ are given in Table 1. These electromotive forces are probably not in error in any case by as much as 0.1 millivolt. The free-energy-decrease is that attending the cell-reaction $\frac{1}{2}H_2$ (1 atm.) $+ \frac{1}{2}Hg_2Cl_2$ (solid) $=$ Hg (liquid) $+ H^+Cl^-$ (at concentration c). The table also contains the values of the increase $(H_2 - H_1)$ in the heat-content of the cell when this reaction takes place at 25°, calculated by the fundamental thermodynamic equation:

$$\frac{H_2 - H_1}{T^2}\,dT = d\left(\frac{F_1 - F_2}{T}\right).$$

TABLE 1

ENERGY-EFFECTS RELATING TO THE REACTION
$\frac{1}{2}H_2$ (1 ATM.) $+ \frac{1}{2}Hg_2Cl_2 = Hg + HCl$ (AT VARIOUS CONCENTRATIONS).

MOLS HCl PER 1000 G. WATER	ELECTROMOTIVE FORCE AT			FREE-ENERGY DECREASE AT			HEAT DECREASE AT 25
	18°	25°	35°	18°	25°	35°	
4.484	0.15759	0.15506	0.15124	15208	14964	14595	25590
1.9278	0.23769	0.23589	0.23304	22937	22764	22489	30480
1.0381	0.27919	0.27802	0.27595	26942	26829	26629	32130
0.7714	0.29654	0.29571	0.29411	28616	28536	28381	32450
0.5095	0.31912	0.31865	0.31765	30795	30750	30654	33070
0.3376	0.33845	0.33836	0.33794	32661	32652	32611	33380
0.1004	0.39764	0.39884	0.40013	38373	38489	38612	34060
0.0333	0.45020	0.45258	0.45557	43444	43674	43963	34370

In Table 2 are given the corresponding values of the free energy of transfer and heat of transfer of 1 HCl from solutions of various concentrations to a solution of the concentration 0.1000 mols HCl per 1000 g. water. These are obtained from the values of Table 1 by direct subtraction (after reducing the values at the concentration 0.1004 so as to correspond to the round concentration 0.1000). In the table are included also values of the free energy of transfer at 18° for concentrations below 0.0333 molal, these having been calculated from Jahn's[2] measurements of the electromotive force of concentration-cells of the type Ag+AgCl, HCl (c_1), HCl(c_2), AgCl+Ag. In the next to last column of the table are given the corresponding values of the activity-coefficients at 18°, calculated by the equation given above on the assumption that at the smallest concentration (0.00167 molal) the activity-coefficient is equal to the ionization-coefficient (0.988) derived from the ratio of the equivalent conductance at that concentration to that extrapolated for zero concentration. The last column of the table gives the values

of this conductance-ratio at 18° at the other concentrations, enabling a comparison of it to be made with the values of the activity-coefficient.

It will be seen from Table 2 that with increasing concentration the activity-coefficient first falls more rapidly than the conductance-ratio, being about 10% smaller than the latter at concentrations 0.1 to 0.5 molal. This shows that at these concentrations there is an error of this magnitude in the common practice of employing in mass-action expressions the conductance-ratio as a measure of the activity of the ions of the acid. The activity-coefficient, moreover, unlike the conductance-ratio, passes through a minimum at about 0.50 molal, and then increases rapidly with the concentration, becoming about equal to that

TABLE 2

ENERGY-EFFECTS ATTENDING THE TRANSFER OF HYDROCHLORIC ACID AND VALUE OF ITS ACTIVITY-COEFFICIENT

MOLS HCl PER 1000 G. WATER	FREE-ENERGY DECREASE AT			HEAT DECREASE AT 25°	ACTIVITY-COEFFICIENT AT 18°	CONDUCTANCE-RATIO AT 18°
	18°	25°	35°			
4.484	23184	23544	24037	8474	2.23	....
1.9278	15454	15744	16143	3591	1.063	....
1.0381	11449	11679	12003	1937	0.864	0.841
0.7714	9775	9971	10250	1619	0.823	0.868
0.5095	7596	7757	7978	999	0.795	0.889
0.3376	5730	5856	6021	684	0.816	0.901
0.1000	0	0	0	0	0.844	0.925
0.03382	−5053	−5167	−5331	−300	0.892	0.955
0.01668	−8224	....	....	....	0.926	0.966
0.01115	−10084	....	....	....	0.943	0.971
0.008324	−11447	....	....	....	0.953	0.976
0.006683	−12472	....	....	....	0.960	0.978
0 005569	−13325	....	....	....	0.967	0.980
0.003334	−15717	....	....	....	0.985	0.985
0.001667	−19031	....	....	....	0.988	0.988

ratio at 1 molal, and attaining at 4.5 molal a value $2\frac{1}{4}$ times as great as that at zero concentration. This large increase is in correspondence with the rapid increase of the vapor-pressure of the acid at high concentrations.

Other exact electromotive force investigations from which ion-activities can be derived have been published by Jahn[2] on potassium chloride and sodium chloride at concentrations between 0.00167 and 0.033 molal and by MacInnes and Parker[3] on potassium chloride between 0.001 and 0.5 molal. The results of the last-named investigators give for the activity-coefficient of potassium chloride the values 0.653 at 0.5 molal, 0.738 at 0.1 molal, and 0.885 at 0.01 molal. The corresponding values

for hydrochloric acid presented in this article are 0.795, 0.844, and 0.945. The value (0.738) for potassium chloride at 0.1 molal is again much smaller (namely about 14% smaller) than the conductance-ratio (0.861).

A more complete description of this research will soon appear in the *Journal of the American Chemical Society*. The preparation of the cells so as to secure constancy and reproducibility of the electromotive force values, the methods of making the measurements, the full experimental data, and thermodynamic calculations from them of other free-energy values will be there presented in detail.

This research has been carried on with the coöperation of Prof. A. A. Noyes and with the aid of a grant made to him by the Carnegie Institution of Washington. The preliminary experiments were made jointly with Dr. Louis Weisberg, and the final measurements with Mr. Frank W. Hall. For all this assistance I wish to express my great indebtedness.

[1] Lewis, *Proc. Amer. Acad.*, **43**, 259–293 (1907); *Zs. physik. Chem.*, **61**, 129–165 (1908).
[2] Jahn, *Zs. physik. Chem.*, **33**, 545–576 (1900).
[3] MacInnes and Parker, *J. Amer. Chem. Soc.*, **37**, 1445–1461 (1915).

EFFECTS OF CENTRIFUGAL FORCE ON THE POLARITY OF THE EGG OF CREPIDULA

By Edwin G. Conklin

DEPARTMENT OF BIOLOGY, PRINCETON UNIVERSITY

Read before the Academy, November 16, 1915. Received, January 22, 1916

If the eggs of the marine gasteropod Crepidula plana are subjected to centrifugal force of approximately two thousand times gravity the yolk is thrown to the distal or centrifugal pole, the oil and other light substances to the centripetal pole, while the nucleus and centrosphere together with most of the cytoplasm occupy the middle zone between the other two. In eggs centrifuged after fertilization and before the first cleavage the yolk zone comprises a little more than three-quarters of the volume of the whole egg, the middle zone a little less than one-quarter and the oil zone about one sixty-fourth, the relative volumes of the three being 49:14:1. In normal eggs of this stage the nucelus centrosphere and most of the cytoplasm lie near the animal pole, but in centrifuged eggs these formative substances may be displaced far from this position, the yolk, for example, being thrown to the animal pole and the protoplasm to the vegetative pole, or these displacements may take place in any other axis. Nevertheless these substances slowly come back to their normal positions provided there is sufficient time for this before the next cell division. However if cell division intervenes

before the egg substances have regained their normal positions there is usually an abnormal distribution of these substances to the two daughter cells and since these substances do not pass through partition walls this abnormal distribution persists throughout later development. In each of the daughter cells, however, the protoplasm goes to the animal pole and the yolk to the vegetative one and the subsequent development is as nearly normal in every respect as is possible, although it is not possible to entirely undo the effects of the earlier dislocation. These results are in substantial agreement with the earlier work of Morgan (1907, '09, '10), Lillie (1906, '09), Conklin (1910, '12), et al., on the development of centrifuged eggs of other animals.

We have in these cases one of the simplest examples of organic regulation in which the organization concerned is merely the polarity of a single cell. The fact that polarity and pattern of organization persist in an egg after cytoplasm, nucleus, centrosphere, yolk and practically all other visible constituents of the cell have been displaced far from their normal positions is most remarkable and mysterious. Is polarity the result of some immaterial influence, some 'entelechy,' which like a divinity 'shapes our ends, rough hew them how we will?' Is it a purely physiological phenomenon, dependent as Child (1911, '12, '13, '14) believes on the rate of metabolism at different poles? Or does polarity persist in some material substance which is not moved by centrifugal force? Investigators are not inclined to close the door to scientific research by a hasty resort to 'entelechy,' and differences in the rate of metabolism or of any other physiological process at two poles, when not due to differences in the environment, must be associated with differences in the material substances at those poles. Consequently it seems necessary to conclude that there is some material substance or relation of parts in these eggs which persists with relatively little change, in spite of the dislocations caused by centrifuging, and which is capable of bringing these substances back to their normal positions when centrifuging ceases, unless they have been isolated in the meantime by the interposition of partition walls.

What is this substance or relation of parts in which polarity persists? Lillie (1906) concludes that polarity is a property of the 'ground substance' of the egg, this substance being 'a fluid which has no filar, reticular or alveolar structure' but yet is 'firmly organized' so that it is not affected by centrifuging. The substances which are dislocated by centrifugal force are mere 'inclusions' in this 'ground substance;' consequently eggs in which these inclusions are forced to occupy abnormal positions are still capable of normal development since their 'ground

substance' remains unaltered. It is evident from Lillie's use of this term that he means it to include what is commonly called 'protoplasm' as contrasted with the 'inclusions' or metaplasm.

In eggs which contain relatively little yolk it may appear possible to identify this 'ground substance' with the general protoplasm, but in the eggs of Crepidula, where the yolk constitutes more than three-quarters of the whole, the protoplasm, including under this term cytoplasm, nucleus and centrosphere, is moved bodily from one pole to another when the yolk is displaced from its normal position. If there is a 'ground substance' here which is not moved by centrifuging it must constitute a relatively small part of the general protoplasm of the egg.

There is good evidence that this is indeed the case and that while the greater part of the cytoplasm is more or less free to move under the influence of centrifugal force there is a portion which is denser and more resistent which is not stratified by centrifugal force. Indeed as long as an egg remains alive this stratification is never complete and the boundaries of the zones are neither plane surfaces nor are they sharply delimited as is the case in dead eggs which have been centrifuged. In Crepidula this denser and more resistant part of the protoplasm is found in the nucleus and centrosphere, in a thin peripheral layer which retains its position as long as the egg remains unbroken, and in a framework of strands which runs through the cell and connects the nucleus and centrosphere to the peripheral layer. During non-divisional stages these strands are slender and less resistant and may be stretched or distorted but are rarely broken. They hold the centrosphere and nucleus in a definite relation to each other and to the cell axis, the centrosphere always lying on the side of the nucleus toward the animal pole and next to the free border of the cell. In whatever axis the egg may be centrifuged and however far the centrosphere and nucleus may be carried from the animal pole, the centrosphere always retains this position on the animal pole side of the nucleus and often can be seen to be connected with the peripheral layer by strands of denser protoplasm.

During cell division the strands radiating from the centrosome are stronger and more resistant than during non-divisional stages so that it is much more difficult to move the mitotic figure than it is to shift the resting nucleus and centrosphere. Indeed it is practically impossible to move this figure when once the astral radiations are well developed without destroying the power of further development. On the other hand if nucleus and centrosphere are displaced from their normal positions during non-divisional stages and are prevented from returning by prolonged centrifuging they undergo division in these new positions

and give rise to cells which are more or less removed from their normal places. Thus polar bodies and ectoderm cells may be caused to form at the vegetative pole or at any other point on the surface of the egg instead of at the animal pole; nevertheless the protoplasmic portions of the yolk-containing cells return to the animal pole when centrifuging ceases unless otherwise prevented. In some instances in which centrifuging took place in the two cell stage the normal positions of protoplasm and yolk are regained in one of the cells but not in the other. All such cases indicate that it is difficult but not absolutely impossible to change the polarity of eggs and cleavage cells, and that the persistence of polarity in centrifuged eggs and the restoration of dislocated parts to their normal positions is connected with a somewhat resistent framework of protoplasmic strands which preserve the relative positions of nucleus and centrosphere in the cell axis.

Child, C. M., Studies on the Dynamics of Morphogenesis, etc. I–V, VII–VIII. *J. Exp. Zool.*, 10, 11, 13, 14, 16, 17 (1911–1914).

Conklin, E. G., Effects of Centrifugal Force on the Organization and Development of the Eggs of Fresh Water Pulmonates. *Ibid.* 9 (1910).

Lillie, F. R., Observations and Experiments Concerning the Elementary Phenomena of Embryonic Development in Chaetopterus. *Ibid.* 3 (1906). Polarity and Bilaterality of the Annelid Egg, Experiments with Centrifugal Force. *Biol. Bull.*, 16 (1909).

Morgan, T. H., Cytological Studies on Centrifuged Eggs. *J. Exp. Zool.*, 9 (1910).

THE EMISSION QUANTA OF CHARACTERISTIC X-RAYS

By David L. Webster

JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY

Received by the Academy, January 15, 1916

At the meeting of the American Physical Society last April, Duane and Hunt announced that X-rays of any given length would be excited as a part of the general radiation from a tungsten target only if the potential applied to the tube was enough to give an electron a kinetic energy as large as the Planck quantum of that wave length. This law was confirmed and extended to a potential of 100 kv by Hull. Off hand one would expect it to apply to characteristic rays also; but, since some work of Whiddington suggests an exception here, it seemed desirable to test such rays with the spectrometer.

This was done with a rhodium target in a tube which Dr. Coolidge very kindly had made with one of his hot wire cathodes. The potentials were supplied by a storage battery of 20,160 cells, and could be measured to about 1%. Neighboring potentials could be compared to about 0.1%.

The part of this work dealing with the general radiation of rhodium confirms Duane and Hunt's law exactly, and leads to a value of h of 6.52×10^{-27} erg sec, taking e as Millikan's value 4.774×10^{-20} esu. and the calcite spacing as Bragg's value 3.025 A. This lies above the values given by Duane and Hunt[1] and Planck[2] and below the values given by Millikan and Hull.[3] If the spacing of the calcite planes is recomputed on the basis of Millikan's e, it is 3.03 A, and the value of h is 6.53×10^{-27} erg sec. The wave lengths and h values given with the graphs in this paper are computed from this value of the spacing.

As a consequence of Duane and Hunt's law, if the spectrometer is set to receive one of the characteristic lines, some radiation is received as

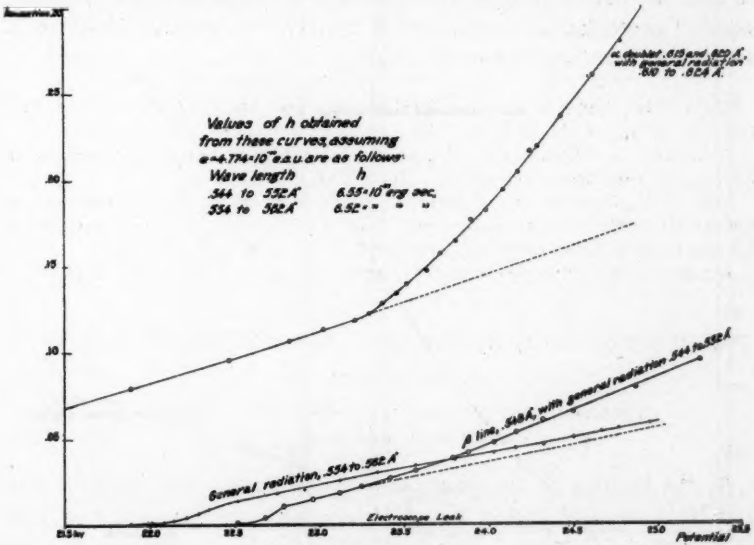

Values of h obtained
from these curves assuming
e=4.774×10⁻¹⁰ e.s.u. are as follows:
Wave length h
.544 to .532 A 6.55×10⁻²⁷ erg sec.
.534 to .582 A 6.52×10⁻²⁷ " "
α doublet, .615 and .622 A,
with general radiation
.600 to .624 A.
β line, .548 A, with general radiation .544 to .532 A
General radiation, .534 to .582 A
Electroscope Leak
Potential

Fig. 1

soon as the quantum potential is reached; but it is found that the intensity is at first only what one would expect from the general radiation at that wave length of no lines existed. With these rays, however, at a definite higher potential the intensity suddenly starts to increase more rapidly than before. This point therefore is the lowest potential at which any characteristic lines appear.

This effect is shown in figure 1, the upper graph giving the intensity as a function of potential for the α doublet. The first part of the curve and its dotted continuation are due to general radiation alone, while the part to the right of the corner is due to the conbined general radiation

and α doublet. The critical potential is very sharply defined, and has a value 23.3 kv, while the quantum potential, at which radiation of that wave length first begins to appear, is 20.05 kv. The same effect is shown by the β line, as one may see by comparison of its intensity-potential curve (fig. 1) with that of the general radiation of a neighboring wave length. The effect is less pronounced because the line is weaker than α. The critical potential, as nearly as one can tell, is the same in both cases.

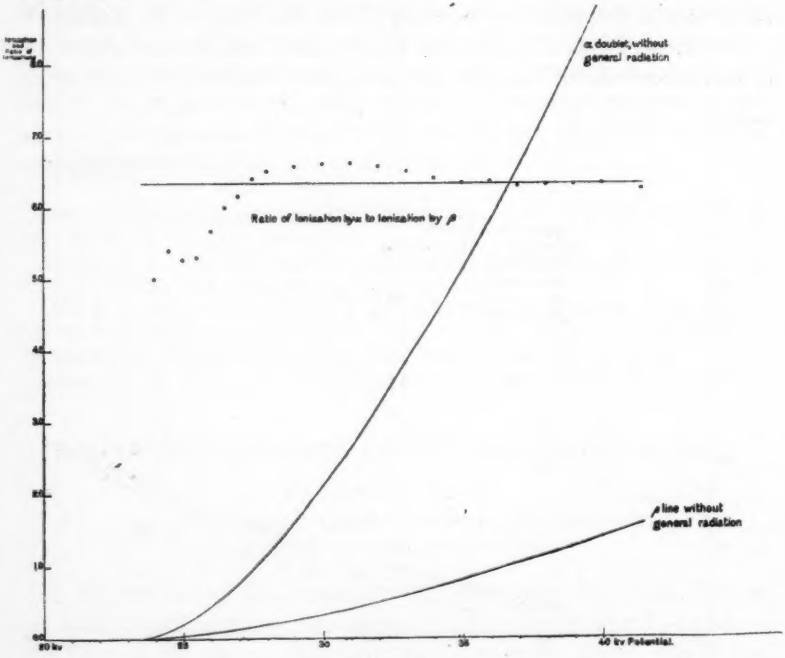

α doublet, without general radiation
Ratio of ionization by α to ionization by β
β line without general radiation
20 kv
30
40 kv Potential

Fig. 2

To prove that the corner in the α curve really does mean the first appearance of the α lines, photographs of this part of the spectrum have been taken at different potentials above and below the critical one. These show that the ratio of the intensity of the lines to that of the background, as well as the absolute intensity of either, increases rapidly with the potential, and that, while the lines are visible against the background within 1% above the critical potential, they are absolutely invisible below it.

The next point to consider is the ratio of intensities of any two lines and its dependence on potential. From numerous photographs with different exposures it appears that the ratio of α_1 and α_2 is constant, as

nearly as one can tell, from the critical potential to the limit of the battery's capacity, at about 42 kv. The ratio of the β line to the α doublet can be estimated only by allowing for the general radiation that appears in the spectrometer with each of them. This can be done, only roughly,

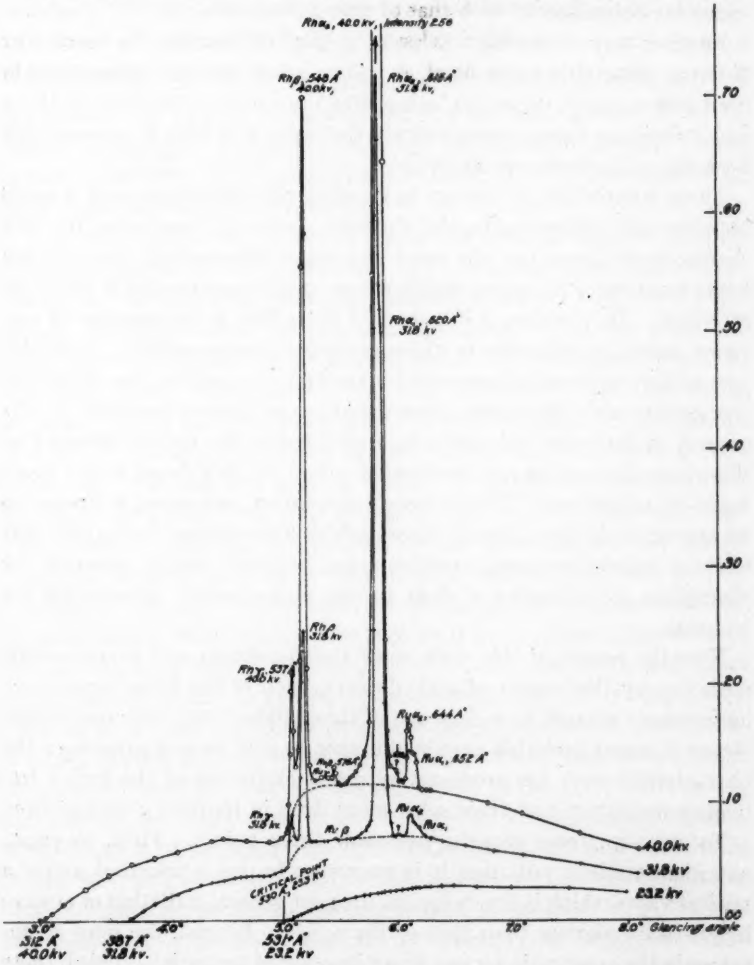


Fig. 3

by the use of the intensity-potential graphs for the combined lines and general radiation and the comparison of them with the graphs for the general radiation at neighboring points in the spectrum. The resulting graphs, for the lines alone, and the graph of their ratio against potential, are shown in figure 2. The constant ratio indicated there differs from

the experimentally determined points by less than the limits of error, which are of course much larger at low potentials than at high. Since it is really a ratio of ionizations in an arbitrary amount of ethyl bromide, no importance can be attached to its absolute value, but its constancy seems very significant.

Another way of testing this point is that of plotting the spectra at different potentials as in figure 3. For various reasons this method is even less accurate than the other, but the ratios of increase of the α and β lines are again approximately the same, and here it appears that even the γ line increases similarly.

These spectra are of interest in showing also the influence of a small impurity of ruthenium in the rhodium target in producing its own characteristic lines; but the most important information they contain is the location of the wave length whose quantum potential is the critical value. Its position, 1.3% short of the γ line, is independent of any errors entering uniformly in these potential measurements. Now this part of the spectrum is known to be marked by a sudden rise of absorptive power with decreasing wave length, and indeed the drop in the general radiation at this point is undoubtedly due to the influence of this absorption on the rays leaving the target, as they do, at a very small angle to its surface. This increased absorption, moreover, is known to be accompanied by a strong characteristic fluorescence, indicating that when a higher frequency oscillator has acquired whole quantum by absorption it undergoes a drop to the characteristic frequencies for emission.

Now the results of this work show that to obtain any characteristic radiation by the impact of cathode rays, each of the latter must have have energy enough to satisfy one of these higher frequency oscillators. Hence it seems probable that in this case as well as in fluorescence the characteristic rays are produced by direct excitation of the higher frequency oscillators and their subsequent drop in frequency on emission.

To sum up, these experiments show three points. First, to excite any characteristic radiation it is necessary to use a potential above a critical value which is the value required for general radiation of a wave length 1.3% shorter than that of the γ line. Second, the lines all increase in the same ratio for any given increase of potential. Third, there is reason to believe that the characteristic rays are always a result of excitation of higher frequency oscillators, as in the case of fluorescence.

[1] Amer. Phys. Soc., April, 1915; *Physic. Rev.*, July, 1915. Recomputed on the basis of Millikan's e their value is 6.50.

[2] *Heat Radiation*, translation by Masius, p. 172, gives 6.415.

[3] Amer. Phys. Soc., Dec., 1915. These values are 6.57 and 6.59 respectively.

THE RESULTS OF INVESTIGATIONS OF THE ECOLOGY OF THE FLORIDIAN AND BAHAMAN SHOAL-WATER CORALS

By Thomas Wayland Vaughan

U. S. GEOLOGICAL SURVEY, WASHINGTON, D. C.

Received by the Academy, January 27, 1916

Many of the most important principles of coral ecology were long ago recognized and clearly formulated by Darwin and Dana. More recently Klunzinger, Pourtalès, Moseley, Alexander Agassiz, Verrill, Stanley, Gardiner, von Marenzeller, Duerden, Wood-Jones, and others have made important contributions.

The coral faunas which live in water less than 25 fathoms deep in coral-reef regions are separable into two subfaunas according to their ability for withstanding violently agitated water. These are (1) the strong, firmly attached, usually massive forms which can withstand breakers and the pounding of the surf; and (2) the weakly attached and branching forms which can survive only in quiet water. The forms requiring quiet water are further subdivisible according to their capacity to resist the deleterious effects of silt. A species of massive growth habit often will also live in quiet water. In some instances the same species of branching coral may be represented in both quiet and rough water, but the colonies in the rough water have shorter and stouter branches, responding to the environment by strengthening their skeletal structures. Massive, large, head-like corals, such as *Orbicella annularis*, form the strong frame-work of the reef while in the interspaces between the heads many colonies of species of smaller size grow and other organisms are present in greater or less abundance. *Acropora palmata*, a species of another growth habit, is an important reef builder in places. It forms ascending fronds which by the thickening of their basal portion become very strong. Two species which live in the quiet water on the flats behind the outer reefs or in the lagoons, are *Maeandra areolata*, which because of its small base of attachment could not remain fixed on the reef, and *Porites furcata*, which because of its fragile branches would be smashed to bits by the rough water of the outer reefs. These two species can exist where the bottom is muddy, as both possess the means necessary for ridding themselves of considerable quantities of silt. *Eusmilia fastigiata* is a species which has a fragile skeleton and requires quiet water, but as it can not endure much silt, it is restricted to areas where the bottom is cleaner. *Porites clavaria* illustrates responsive adaptation to environment, as it lives both on the reef and on the inner flats. The branches of the colonies in the former habitat are

short and stumpy or the colony may be almost massive in growth form, while in quiet water the branches may be decidedly elongated.

The depth to which the more massive forms extend is between 18 and 31 meters, 18 meters is usually about the maximum for vigorous growth, but some of the branching species extend to slightly greater depths. In general the lower depth of the shoal-water coral fauna of the West Indies is about 37 meters, approximating conditions in the Pacific. The precise cause of the limit in depth has not been determined. Each of several possible factors will be discussed.

All the corals with which I have experimented possess the capacity of removing a certain amount of sediment from their surfaces. This is affected by the nonnutrient particles becoming imbedded in mucus and by cilia removing the mucus and the particles from the surface of the tissue. The capacity for cleaning their surfaces varies according to the species, it being lowest among those corals which are most important on the outer reefs—it is low in *Orbicella annularia* and high in *Maeandra areolata*. Some corals, as *Siderastrea radians*, can endure having their surfaces covered with silt for some time. This coral seems to secrete a layer of mucus which lifts the silt above the tissue surfaces and thereby protects them. The branching form of many corals prevents sediment settling on them faster than it can be removed. However, as any coral will be killed by actual burial beneath sediment, corals can not live where sedimentation is rapid; and as sediment accumulates in areas deeper than the base of strong wave action or where currents are weak, it is a factor in limiting the depth to which the littoral fauna can extend.

The mechanism of corals for catching food are as follows: (1) The ectodermal surface is beset with nematocysts, which occur on the tentacles, the oral disk, the column wall, including its downward extension called the edge-zone, and also on the margin of the mesenterial filaments. (2) The entire ectodermal surface is ciliate, the cilia in response to certain stimuli beating toward the oral apertures; in response to others, beating toward the periphery. (3) The outer surface secretes mucus in which particles may be embedded, the mucus moving under the influence of the beat of the cilia toward the oral apertures or toward the periphery, according to the nature of the response to the stimulation. (4) The tentacles are active and effective in capturing food. (5) The mesenterial filaments, which in many species of corals can be extruded through the column walls, in some instances capture food.

Many different kinds of food were offered corals, but they took only animal food; they are entirely carnivorous. The following experiment was tried many times: A piece of diatom mat was placed on one side

of the oral disk and a piece of crab meat on the other. Invariably the crab meat was seized and swallowed; while the diatoms induced no reaction except ultimately to be removed from the surface. No kind of purely vegetable food was taken by any one of the numerous species investigated. However, pieces of plants coated with small animals or soaked in meat juice will be swallowed, and later the vegetal matter ejected.

As the food of corals is purely animal plankton, a decrease in the amount of this food-supply with increasing depth would limit the downward distribution of the shoal water forms, but as I do not know of any quantitative estimates of the amount of animal plankton above and below 20 fathoms in coral reef areas, there is no basis for a positive opinion.

The relation of corals to light was studied. Specimens of 17 species were put into a darkened, light-proof live-car. One of the number was dead at the end of 14 days; 3 others were dead at the end of 28 days; while 11 species survived at the end of 43 days. However, all had become pale, some even colorless, or otherwise showed abnormalities. A natural experiment, which appears conclusive, is afforded by Fort Jefferson wharf. Here corals thrive on all the outer piers where the light is strong but there are none on the central piers where there is perpetual shade. It therefore seems to me that strong light is essential for the vigorous growth of shoal water corals.

Another factor is temperature. Dr. Mayer conducted a series of experiments to ascertain the higher and lower limits of temperature which the common corals around the Tortugas can endure. These indicate that a lowering of the temperature to 13.9°C. would exterminate the principal Florida reef corals, while the most important inner flat corals would survive. He obtained similar results on the corals around Murray Island, Australia.

Dr. H. F. Moore of the U. S. Bureau of Fisheries has communicated to me temperature records made at lighthouses along the Florida reef. These show that vigorous reefs will endure a temperature as low as 18.15° C., the minimum at Carysfort light between 1879 and 1899; but at Fowey Rock, where the minimum drops to 15.6°C. although there are some corals, there is no thriving reef. The species found at the north end of the reef line are those which Dr. Mayer's experiments showed capable of withstanding the lowest temperature. The temperature records for the reef line indicate 18.15°C. as the minimum temperature which a reef will survive—this is 1.85°C. lower than the figure given by Dana. It is not probable that a reef could withstand a continuous

temperature so low as this. Wherever the depth of water is great enough to lower the bottom temperature below 18.15°C., more probably about 21°C., reef corals will not live. This temperature appears to be attained around the Hawaiian Islands within a depth of 183 meters. According to Agassiz's *Three Cruises of the Blake* the bottom temperature in the Gulf of Mexico and Caribbean Sea is usually too low for the growth of reef corals at a depth of 183 meters, and in places it is too low at a depth of 87 meters. Although the possibility of control of the lower bathymetric limit of reef-building corals by decrease in temperature with increasing depth has not been adequately investigated, it appears safe to say that reef corals are usually, if not always, confined by temperature to water less than 180 meters deep.

The four possible factors which tend to limit the downward extent of reef forming corals are as follows: (1) effect of sediment, (2) decrease in supply of small animal plankton, (3) decrease in intensity of light, (4) lowering of the temperature.

The relations of corals to salinity will now be considered. The average salinity of the Tortugas water according to Dole is 36.01%. Seventeen species of the Tortugas corals were kept in a large tank of water with a salinity of 18.28% for 24 hours. All were damaged or killed except *Maeandra areolata*, *Siderastrea radians*, and *Porites astreoides*; but no specimen of 16 species showed any evidence of harm after remaining 48 hours in water of a salinity of 27.87%. Apparently corals would not be hurt if the salinity of the ocean were reduced to about 80% of its present salinity. Although I did not experiment with concentrated sea-water, the studies made by Goldforb and others on the effect of concentrated and diluted sea-water on regeneration in hydroids and in the *Cassiopea* are here pertinent. The combined results of the experiments are in accord with the deductions made by oceanographers and geologists from other data, viz., the ocean is becoming more salt, and it appears that marine organisms are now living in an environment which is considerably below the optimum condition for their existence.

In order to ascertain the amount of atmospheric exposure corals would endure, experiments were made on 16 species, any of which will endure exposure on a glass plate in the shade for half an hour without apparent damage; nearly all will stand an hour without harm, while some will stand 4 hours' exposure under the conditions stated. *Favia fragum*, *Porites clavaria*, and *Porites astreoides* have the greatest capacity for withstanding atmospheric exposure, while that of *Maeandra areolata* and *Siderastrea radians* is almost as great. A number of species withstood exposure on a glass plate in the sun for 1½ hours, the specimens

being badly damaged, but not entirely killed. Although not precisely true, in general the ability to withstand atmospheric exposure is a function of the porosity of the skeleton, the species with the more porous, surviving longer than those with the denser skeletons.

The conditions necessary for vigorous coral reef development may be summarized as follows: (1) Depth of water, maximum, about 45 meters; (2) bottom firm or rocky, without silty deposits; (3) water circulating, at times strongly agitated; (4) an abundant supply of small animal plankton; (5) strong light; (6) temperature, annual minimum not below 18°C.; (7) salinity between about 27% and about 38%.

In the experiments on rearing corals, the planulae were removed with a pipette from the vessel containing the parent colony to a jar on the bottom of which was a terra-cotta disk. Although the planulae will live a long time, even settle in stale water, kept at the proper salinity, it is better to change the water at least once a day. To change the water, siphons were used, a fine-mesh bolting cloth bag having been tied on the end within the jar, so as to prevent the escape of the planulae; while clean water was added through siphons from jars placed at a higher level.

Because of its bearing on the possibility of the distribution of coral species by oceanic currents, it is highly important to know the duration of the free-swimming larval stage. Observations were made on four species. The range was from 2 to 23 days. Should an ocean current have a velocity of 3 knots per hour, in 23 days planulae might be carried 1656 knots; at 2 knots per hour, 1104 knots; at 1 knot per hour, 552 knots. It is known that every species of shoal water coral in the Bermudas is found in Florida and the West Indies; while not only is the Hawaiian fauna Indo-Pacific in its facies, but several of the species (at least 4) also occur on the east coast of Africa or in the Red Sea. I seriously doubt any part of the Hawaiian fauna being peculiar to those Islands. The clue to the cause of the wide distribution of living coral species is given by the possibly long duration of the free-swimming larval stage.

The growth rate of corals was determined by planting planulae which attached themselves in the laboratory, by measuring colonies, from planulae which settled on collectors in a known season, by measuring colonies cemented to disks and fixed on the heads of stakes driven into the sea-bottom, and by measuring colonies naturally attached. The plantings around the Tortugas were made on the reef off Loggerhead Key and on the outside of Fort Jefferson moat wall, while records were made on colonies growing naturally attached at the two stations men-

tioned, on the piers of Fort Jefferson wharf, and in Fort Jefferson moat. The observations in the Bahamas were made on artificially planted and naturally attached colonies at the east end of South Bight, Andros Island. The Florida corals were measured annually; those in the Bahamas were measured in 1912 and again in 1914. The average growth rate for each species at each station has been computed. The size of the colonies of all species of corals seems limited, but some attain large dimensions, 2 to 3 meters or even more in diameter, and nearly as much in height, while other species are adult when a diameter of 35 to 50 mm. has been reached. *Favia fragum* and *Maeandra areolata* are instances of species which grow relatively rapidly for the first 2 to 4 years, after which they grow more slowly. *Orbicella annularis* and *Maeandra strigosa* are not so limited in size. Branching corals grow more rapidly than massive species; while of the former, the growth rate of species with perforate, loose-textured skeletons is more rapid than that of species with dense skeletons. In general the more massive and the denser the corallum, the slower the growth; while the more ramose and the more porous the skeleton, the more rapid the growth.

There is no average growth rate for corals generally speaking, as growth rate varies from species to species, and varies for the same species according to local environmental conditions. Here it may be said a colony of species of reef coral in a lagoon, if protected from sediment, may grow more rapidly than a colony of the same species does on the reef. The limitation of reef corals so largely to the outer edges of platforms is determined primarily by purity of water, i.e., freedom from silt, and by the more uniform temperature.

In order to estimate the rate at which a reef will grow, the upward growth rate of the true reef-forming species must be taken. The upward growth rate of *Orbicella annularis*, the principal builder of the Pleistocene and living reefs in Florida and the West Indies, is from 5 to 7 mm. per year, according to station. At 6 mm. per year, it would form a reef 150 feet thick in 7620 years; at 7 mm. per year it would build the same thickness of rock in 6531 years. *Acropora palmata*, which grows more rapidly, might build a similar thickness in 1800 years. The growth of corals in the Pacific appears to be more rapid and according to Stanley Gardiner they might build a reef 150 feet thick in 1000 years. The investigation of the growth rate of corals shows that any known living coral-reef might have formed since the disappearance of the last continental ice sheets.

(This summary is published by permission of the President of the Carnegie Institution of Washington and of the Director of the U. S. Geological Survey.)

CAMBRIAN TRILOBITES

By Charles D. Walcott

SMITHSONIAN INSTITUTION, WASHINGTON, D. C.

Received by the Academy, January 31, 1916

The writer has assembled data to aid in clearing up some of the problems of formations of the Appalachian region by a careful comparison of portions of their contained faunas with those of the Mississippi Valley, the Cordilleras, and other localities. No thorough study and comparison of many genera of the Cambrian faunas has been made, though collections from many outcrops have been in the writer's possession for years, awaiting the opportunity to make these studies so necessary in his work on the Cambrian Trilobites.

Two new families of trilobites are proposed, Menomonidæ and Norwoodidæ, and seven new genera: *Menomonia*, *Millardia*, *Dresbachia*, *Norwoodia*, *Saratogia*, *Vanuxemella*, and *Hanburia;* 46 new species and three new varieties are described, with 19 earlier described species and several genera. One of the marked features of the paper is the description of a number of genera of the order Proparia: *Menomonia*, *Millardia*, *Dresbachia*, and *Norwoodia*. These, taken in connection with the genus *Burlingia*, described in a previous paper (Cambrian Trilobites, *Smithsonian Misc. Coll.*, **53**, No. 2, 1908, p. 14) establish the existence of a strong group of the order in Cambrian time.

The stratigraphic position of the Weeks formation is changed from Middle to Upper Cambrian, and the problem of whether the Conasauga formation of the Coosa Valley and adjoining areas shall be restricted to the Upper Cambrian, and the Middle Cambrian beds there given a formation name, is left for further detailed study.

The discussion and comparison of the *Crepicephalus* group of trilobites is particularly interesting, including a comparison of 17 different species, 10 of them new species, 3 new varieties, and two undetermined species. The five plates of illustrations of this large trilobite also present many new and interesting features of the animal, now so long extinct.

The details of this investigation with fifteen plates continuing 280 figures will be found in *Smithsonian Misc. Coll.*, **64**, No. 3, 1916.

THE MINUTE STRUCTURE OF THE SOLAR ATMOSPHERE

By George E. Hale and Ferdinand Ellerman

MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Received by the Academy, January 10, 1916

During a total eclipse of the sun, when the light of the disk is completely cut off by the moon, the solar atmosphere is momentarily revealed. The exceedingly faint corona, extending millions of miles into space, can be seen only at such times. But the more brilliant chromosphere, the comparatively shallow atmosphere of luminous gases which completely encircles the sun, and the prominences which rise out of it, can be observed on any clear day with the aid of a spectroscope. First applied to this purpose in 1868, the spectroscope has yielded a large store of information regarding the number, distribution, and nature of the prominences and the structure of the upper chromosphere, as seen in elevation at the sun's limb. It has also permitted the observation of certain phenomena of the solar atmosphere in projection against the disk, but on account of the brilliant background, only their general outlines can be thus detected. In order to study their details we must have recourse to the spectroheliograph.

FIG. 1. DIRECT PHOTOGRAPH OF SUN-SPOT GROUP, 1915, AUG. 7, SCALE: SUN'S DIAMETER =24 CM. (Negative reproduction

With this instrument, first applied to the investigation of the solar atmosphere in 1892, a large number of new phenomena have been brought to light. The spectroheliograph may be briefly described as a moving spectroscope, driven at a uniform rate across the solar disk, and admitting to the photographic plate through a narrow slit the light of a single spectral line. Thus monochromatic images of the solar atmosphere, showing the otherwise invisible clouds of hydrogen or of calcium, iron, or other vapor (the flocculi) are recorded permanently for study. In a region on the solar image, for example, where direct observation shows nothing, or perhaps a group of sun-spots, the spectroheliograph may disclose extensive phenomena of great interest in the solar atmosphere (compare fig. 1, a direct photograph of a group of sun-spots, with fig. 5, Plate I, showing two spectroheliograms of the hydrogen flocculi above and surrounding the group). The purpose of this paper is to com-

pare the structure thus revealed at various levels in the solar atmosphere with that of the lower-lying photosphere and sun-spots.

Langley's well-known paper[1] 'On the Minute Structure of the Solar Photosphere,' illustrated with his unrivalled drawings, is still our best source of information regarding the structural details of the photosphere and sun-spots.[2] After referring to the spots and faculae, Langley goes on to say:

> On attentive examination it is further seen that the surface of the sun everywhere . . . . is not absolutely uniform, but is made up of fleecy clouds, whose outlines are all but indistinguishable. . . . . Under high powers used in favorable moments, the surface of any one of the fleecy patches is resolved into a congeries of small, intensely bright bodies, irregularly distributed, which seem to be suspended in a comparatively dark medium. . . . .

These bright bodies, called 'rice-grains' by Stone, were found by Langley to average from 1″.4 to 2″.6 between centers, the distance decreasing with increase of telescopic power.

"In moments of rarest definition I have resolved these 'rice-grains' into minuter components, sensibly round, which are seen singly as points of light, and whose aggregation produces the 'rice-grain' structure. These minutest bodies, which I will call 'granules,' it will appear subsequently can hardly equal 0″.3 in diameter, and are probably less." In the two squares near the lower right-hand corner of the drawing (fig. 2, Plate I), the granules are the minute bodies forming the not very definite clusters or 'rice-grains.' The squares are 11″.6, corresponding nearly to 5000 miles, on a side. From a careful estimate of the size and number of the granules and 'rice-grains,' Langley concludes that "the properly luminous area is less than one-fifth of the solar surface."[3]

The sun-spots shown in the same figure combine details from several different spots, but accurately indicate the general character of the structure, which is described by Langley as follows:

"The penumbra is all but wholly made up, as it appears on a first examination, of cloud-forms whose structure makes them seem like fagots or sheaves of some elongated objects." Under the highest powers "the penumbra is resolved into 'filaments' of extreme tenuity, which by their aggregation make the 'thatch' (a term used by Dawes), just as the minute granules of the photosphere compose the 'rice-grains'." . . . . "It seems to me that there is no room for doubt, that 'filaments' and 'granules' are names for different aspects of the same thing, that filaments in reality are floating vertically all over the sun,

their upper extremities appearing at the surface as granules; and that in the spots we only see the general structure of the photosphere, as if in section, owing to the filaments being here inclined."

Referring later to the extremely fine filaments, estimated at not over 0".03 in diameter, which are sometimes seen on the umbra, Langley remarks on their resemblance to the filamentous structure depicted in spectroscopic drawings of the chromosphere. To this resemblance, which is shown by recent observations to extend to many additional phenomena, we may now direct our attention.

It has long been known that the surface of the chromosphere, as seen at the sun's limb with a spectroscope, commonly appears as a series of slender filaments like blades of grass, supposed by Secchi to correspond with the grains of the photosphere. To study the structure of these filaments in projection against the disk we may utilize the spectroheliograph, which permits their cross-sections to be photographed at several different levels. Thus, by setting the camera slit on the H_1 or K_1 line, which represents the low-lying calcium vapor, we can determine the size and form of the cross-section at a level below that seen visually in the chromosphere at the limb. The calcium lines H_2 and K_2 represent a somewhat higher level, illustrated in figure 3, Plate I. At this elevation the minute structure is similar to that of the photosphere, but the average size of the small bright flocculi is greater, if we may judge from a comparison with the grains in Langley's drawing, reproduced in figure 2 on the same scale. The smallest calcium flocculi photographed, however, are less than 1" in diameter, and thus do not differ greatly in size from the average photospheric grains.[4]

Spectroheliograms made with light from the center of the $H\alpha$ line of hydrogen depict a still higher level, shown on the same scale in figure 4, Plate I, and on a smaller scale in the stereoscopic picture (fig. 5, Plate I). Figure 4 is enlarged to a scale of nearly a meter to the sun's diameter from an excellent photograph made under almost perfect seeing with the new 13-foot spectroheliograph, recently built in our instrument shop for the 60-foot tower telescope on Mount Wilson. The spectrum is that of the first order of a grating by Anderson, used with a plane mirror at such an angle as to give a dispersion of 3.6 mm. to the angstrom at $H\alpha$. As the camera slit transmits only the central part (about half the width) of the $H\alpha$ line, a high level in the chromosphere is represented. The diameter of the solar image at the focal plane of the 60-foot tower telescope is 17 cm., so that the scale of figure 4 corresponds to an enlargement of $5\frac{1}{2}$ diameters.

For some physiological cause it is difficult or impossible to convey a

correct impression of the hydrogen flocculi from positive reproductions of spectroheliograms. For this reason the stereoscopic picture is reproduced as a negative, and in this the bright structure gives a fairly good idea of the appearance of the absorbing hydrogen (corresponding in reality to *dark* flocculi) on the original negative. Figure 4, in order to be directly comparable with figure 2 and figure 3, is reproduced as a positive, and the flocculi shown by it are therefore the dark regions, which occupy a somewhat smaller total area than the brighter spaces between them. Langley estimated that the bright 'rice-grains' cover less than one-fifth of the total area of the photosphere. The corresponding ratios for the calcium (H_2) flocculi range from 10%, when only the smallest and brightest flocculi are included, to 30% and more when the fainter flocculi are also measured. In the case of hydrogen ($H\alpha$) the ratios vary from 40 to 60%, depending upon the limit of darkness adopted for the faintest objects included. These results are derived, moreover, from only a small number of measures of minute flocculi in thinly occupied areas, and are subject to much uncertainty because of the very wide range of intensity of the flocculi, which renders it difficult to distinguish the less conspicuous ones from the background. The ratios may therefore be considerably modified as the result of a special study of this question, which is now in progress.[5]

The smallest of the dark hydrogen flocculi shown in figure 4 are about 2″ in diameter, or approximately twice the diameter of the smallest calcium flocculi shown in figure 3 at the H_2 level.

The foregoing results, in harmony with those of Langley, Secchi, and Evershed, support the view that the photosphere and the gaseous atmosphere above it are formed of columns of hot gases, rising by convection from the interior of the sun. At the photospheric level precipitation may occur of any materials refractory enough to withstand the high temperature, or if these substances do not exist, the conditions may be such as to cause the gases to emit a continuous spectrum. Higher up, where calcium and hydrogen persist, the radiation of the H and K lines in the gaseous columns still exceeds that of the intervening regions, and bright calcium flocculi are consequently recorded. At the still higher level represented by the $H\alpha$ line, the reduced temperature causes the hydrogen to show its presence mainly by absorption, so that the hydrogen flocculi are darker than the background of generally diffused gases.[6] The stereoscopic picture (fig. 5) permits this hypothesis to be further tested.

Helmholtz estimated the minimum angle between two objects just separated by the unaided eyes to be one minute of arc, but Pulfrich has

shown that stereoscopic vision can be obtained when this angle is considerably smaller. Enlargement of the image by a telescope, and the effect of increased inter-ocular distance realized by combining two photographs of a rotating object like the sun, permit the angle to be reduced to surprisingly small values. Thus even on Helmholtz's basis two photographs of hydrogen flocculi on the central meridian of the sun, taken only five minutes apart at the focus of the 60-foot tower telescope, can be combined to give stereoscopic relief. At a distance of 45° from the central meridian this interval must be increased to nearly 7 minutes, but further magnification of the image can be utilized to reduce the minimum time for this region to five minutes or even less. A short time interval serves very well near the limb, in spite of the smaller displacement, because of the enhanced effect of relief due to the curvature of the sun.

The accompanying stereoscopic picture represents the hydrogen flocculi above and surrounding a southern group of sun-spots about 45° west of the central meridian on August 7, 1915. The two exposures were made at 6h. 19m. and 6h. 26m. a.m., respectively, and were thus separated by a time interval of 7 minutes. The enlargement from the original negative is 1.4 diameters, corresponding to a solar image about 24 cm. in diameter. The conditions necessary to secure stereoscopic vision are thus fully realized. It should be noted, however, that while the appearance of relief is probably genuine, so far as the chief elements of structure are concerned, certain false effects are present due to slight distortion of the images. These have the appearance of a horizontal depression running centrally across the picture, between two rounded ridges, on which lie the fringes of distended flocculi above and below the axis of the spot-group. Other false effects may arise in the case of individual details which change materially in form between the exposures. These are few in number, however, and may be readily detected.

In examining the photograph, one is struck by the general resemblance to Langley's drawings of sun-spots and the photosphere (fig. 2). While the difference in scale between the minute phenomena depicted by Langley and the coarser details of the higher atmosphere must be borne in mind, the resemblance can hardly be devoid of significance. As we have already seen, the minute hydrogen flocculi, in undisturbed regions away from spots, are granular in appearance, though larger than the minute grains of the photosphere. At some distance from the spot group (near the margin of the picture) they give place to slender filaments, extending toward the axis of the group as the much finer penum-

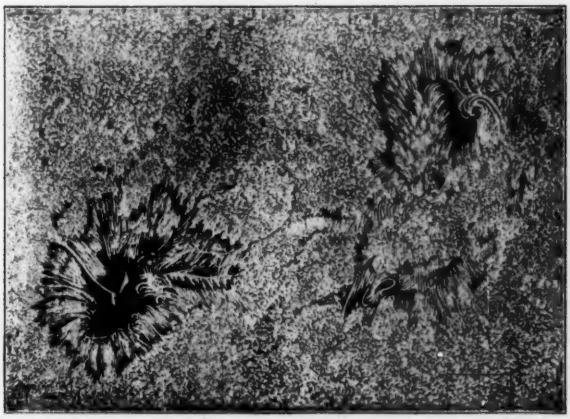


FIG. 2. MINUTE STRUCTURE OF PHOTOSPHERE AND SUN-SPOTS (LANGLEY) SCALE: SUN'S DIAMETER=94 CM. (*Amer. J. Sci.*, 1874)

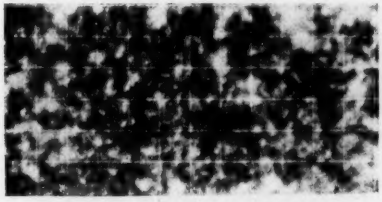


FIG. 3. MINUTE STRUCTURE OF CALCIUM (H_2) FLOCCULI. SCALE: SUN'S DIAMETER=94 CM. (From the Chicago University Press)

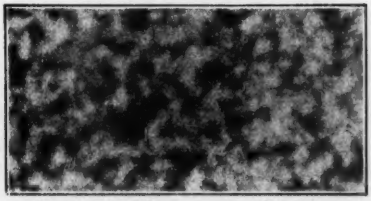


FIG. 4. MINUTE STRUCTURE OF HYDROGEN ($H\alpha$) FLOCCULI. SCALE: SUN'S DIAMETER =94 CM.

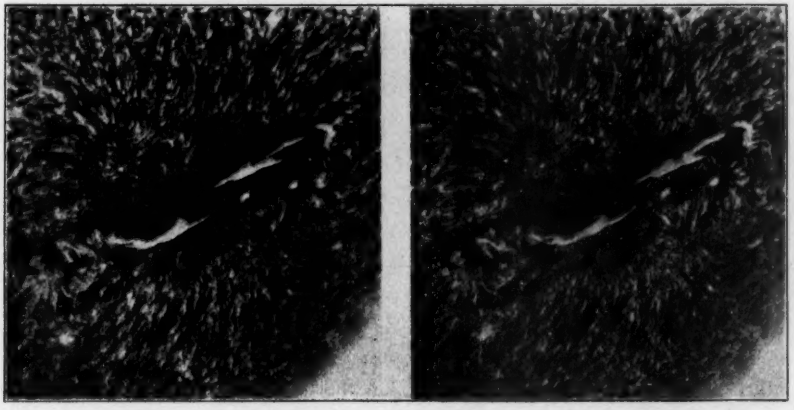


FIG. 5. STEREOSCOPIC PHOTOGRAPH OF HYDROGEN ($H\alpha$) FLOCCULI, 1915. AUG. 7, $6^h\ 26^m$ AND $6^h\ 19^m$ A.M., P.S.T. SCALE: SUN'S DIAMETER=24 CM. (Fig. 5 is a negative reproduction)

bral filaments extend toward the umbra of a sun-spot in Langley's drawing. On either side of this axis, as defined by the long dark flocculus (here bright) which appeared later as a prominence at the sun's limb, they stop abruptly at the edge of a region of honeycomb structure, out of which the long prominence rises like a high ridge. This prominence is shown by the original negatives to be composed of slender filaments, in some parts parallel for long distances, in others apparently intertwined. At its western (right-hand) extremity the prominence curves sharply in a clockwise direction toward the upper spot near the western end of the group (fig. 1), indicated at this level only by a minute white dot, much smaller than the direct image. The extreme western spot, though perhaps the largest in the group, is partially obscured in figure 5, though the hydrogen flocculi may be seen curving toward it. The neighboring spot to the east, however, is plainly visible, and its effect upon the long ridge-like prominence is shown by a second clockwise twist.

The next large spot in the group is clearly seen in the stereoscope, but its smaller companions to the east are obscured by the extensive bright hydrogen flocculi which cover much of the central part of the image, producing the apparently dark background near the middle of the negative print. It should be added that the three large spots just mentioned were shown by spectroscopic observations to be of the same magnetic polarity, indicating that their vortices were rotating in the same direction. This is in harmony with the clockwise curvature of the hydrogen flocculi shown by the stereoscope to be flowing toward each of the spots.[7]

Further to the west (above the center of fig. 1), is a bipolar spot-group, over which the hydrogen vortex is beautifully shown in figure 5 for the western (clockwise) member of the group, though the vortex above the eastern spot is less obvious. The crater-like depression at the center of the western vortex is plainly visible in the stereoscope, which also brings out the bright (dark in fig. 5) star-like boundary surrounding this member of the pair. The magnetic polarity of this spot was the same as that of the three already mentioned.

The two eastern spots near the lower left corner of figure 1 form another bipolar group, but both are partially or wholly obscured in figure 5, and the vortex structure about them is not well defined. At this end of the long ridge-like prominence some divergent filaments rise to a considerable height. These present a remarkable wave-like form in spectroheliograms taken on August 9. It is interesting to note that the highest parts of the prominence and all of the other high level phenomena are dark (bright in fig. 5), as would be expected on the

hypothesis that they result from the absorptive effect of the cooler hydrogen.

It may be added that many spectroheliograms previously and subsequently obtained, especially when this region was carried to the west limb by the sun's rotation, fully bear out the above interpretation of the stereoscopic effect, which is in harmony with the conclusions already stated. Thus they confirm the usefulness of the stereoscopic method, and further substantiate the view as to the nature of these long dark flocculi (called 'filaments' by Deslandres) which we expressed when we first detected them with the $H\beta$ line at the Yerkes Observatory in 1903[8] and again when we photographed them with $H\alpha$ on Mount Wilson in 1908.[9]

We have shown in this paper that the minute structure of the quiescent solar atmosphere resembles that of the photosphere. In disturbed regions, the small granular elements (minute flocculi) are replaced by numerous slender filaments, lying side by side, and recalling the structure of the penumbra in sun-spots. While these results appear to support the hypothesis that the solar atmosphere consists of parallel columns of ascending and expanding gases, which are drawn out horizontally in spot penumbrae and in disturbed regions of the chromosphere, such questions as the dimensions of the columns and the direction of motion and velocity of the vapor in sun-spots and in the atmosphere about them are reserved for subsequent discussion.

A full account of this work, with additional observations and photographs, will be published in the *Astrophysical Journal*.

[1] *Amer. J. Sci.*, Ser. 3, **7**, 87 (1874). Figure 2, Plate I, is taken from this article.

[2] This remark applies to visual observations. The valuable photographic results of Janssen, Hansky, Chevalier and others will be discussed in another paper.

[3] Chevalier, from a series of measures of the bright grains on his excellent photographs of the photosphere, concludes that they cover approximately one-third of the solar surface; *Ann. Obs. Zô-Sè*, **8**, C 20 (1912).

[4] We have shown in a previous paper (*Pub. Yerkes Obs.*, **3**, Part I, p. 14) that the average cross-section of the calcium flocculi at the lower (H_1) level is apparently smaller than at the H_2 level. Thus the calcium vapor probably expands as it rises.

[5] Figure 3, which is taken from our paper cited above, does not give a correct impression of the area occupied by the minute calcium flocculi, as several larger aggregations are present. In determining the size of the minute flocculi, the instrumental conditions, as well as the seeing, must also be taken into account.

[6] The bright eruptive flocculi of hydrogen are not in question here.

[7] The direction of rotation of sun-spot vortices will be discussed in another paper.

[8] *Pub. Yerkes Obs.*, **3**, part I, p. 21.

[9] Solar Vortices. *Contrib. Mt. Wilson Solar Obs.*, No. 26, *Astrophys. J.*, **28**, 100 (1908).

MONOCHROMATIC PHOTOGRAPHY OF JUPITER AND SATURN

By R. W. Wood

DEPARTMENT OF PHYSICS, JOHNS HOPKINS UNIVERSITY

Received by the Academy, January 28, 1916

Photographs of the moon, which I made several years ago by means of ultra-violet light, revealed the existence of an extensive dark area surrounding the crater Aristarchus, which did not appear on plates made by means of yellow light, and could not be detected visually. Control experiments, made in the laboratory, made it appear probable that this deposit consisted of sulphur or sulphur bearing rock.

It appeared to me probable that even more interesting results would be obtained in the case of the planets, especially Mars, and preliminary experiments were carried on during the past summer at my East Hampton Laboratory with a horizontal telescope of 56 feet focal length and 16 inches aperture, figured by Mellish, and nickel plated by the method described in the *Astrophysical Journal* for October.

A deposit of nickel is necessary for the reason that silver reflects only 4% of the light in the region of the spectrum utilized, which lies between wave-lengths 3000 and 3300. This mirror was utilized in conjunction with a large Gaertner coelostat, which was placed at my disposal by the Naval Observatory. The mirror of this instrument was replaced by a 16-inch flat, also nickel plated, and photographs were made of the moon and Jupiter by means of infra-red, yellow, violet and ultra-violet light. Notwithstanding the fact that a moving plate holder, provided with eye-pieces for accurate following, was used, it was found difficult to secure sufficiently sharp definition, owing to the rather rapid periodic drift of the image which was very difficult to follow. The seeing moreover was not very good, and on the few nights when it was fair the mosquitoes were bad. Much valuable experience was gained however in the use of the monochromatic ray filters.

In the earlier experiments on the moon, I used, for the ultra-violet ray filter, a rather thick deposit of metallic silver on uviol glass. This was abandoned in the present work, owing to its great opacity even for the ultra-violet region transmitted, and a rectangular glass cell, closed with plates of uviol glass, and filled with dense bromine vapor, used in its place. Such a cell, when used with an ordinary (i.e., not isochromatic) plate, gives a photographic image formed by wave-lengths below 3500 exclusively. By combining it with a thin layer of a very dilute solution of chromate of potash, the transmission is practically

the same as with the silver film, while the time of exposure is only about one-tenth as long.

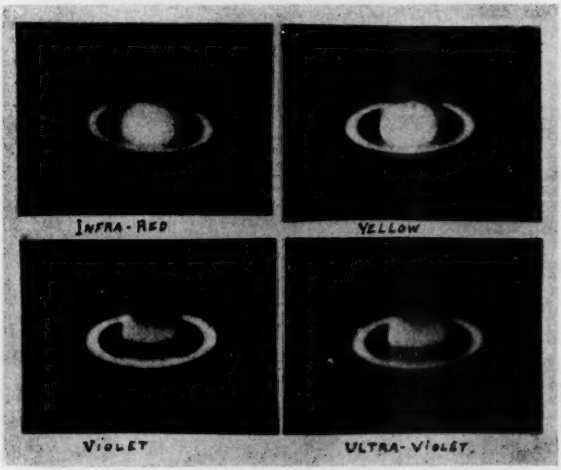
In October, through the courtesy of Dr. G. E. Hale, Director of the Mt. Wilson Observatory, the 60 inch reflector was placed at my disposal for four nights, and by working continuously from sunset to dawn, I was able to secure a very complete set of pictures of the moon in yellow and ultra-violet light and some forty negatives of Jupiter and Saturn in infra-red, yellow, violet and ultra-violet light. The telescope was used at the eighty foot focus and as the mirrors were silvered, the region of the spectrum utilized for the ultra-violet photography was slightly less refrangible than in the earlier work. Preliminary experiments had been made on the spectrum of sunlight after three reflections from silver and transmission through the bromine cell. Full data regarding these matters will be published shortly in the *Astrophysical Journal*.

Extremely interesting results were obtained in the case of Saturn. The infra-red picture, taken by wave lengths above 7200, showed the ball of the planet almost devoid of surface markings, the merest trace of the faint narrow belts appearing. The photographs made with the yellow screen showed the belts distinctly, giving about the same impression as visual observations. On the plates made with the violet ray-filter (transmission 4000–4500), a very broad dark belt surrounded the planet's equator, occupying the region of the planet which was brightest in yellow light. In addition to this dark equatorial belt a dark polar cap of considerable size appeared in the pictures. So different were the two pictures that, were it not for the ring, it would be difficult to believe that they represented the same object. In ultra-violet light the appearance was much the same, but the dark belt was not quite so wide, the bright region between the polar cap and the belt being distinctly broader.

Photographs made with the four monochromatic filters are reproduced. Two hypotheses suggested themselves in explanation of the dark belt. We may be dealing with a fine mist or dust which forms an extension of the crepe ring down to the ball of the planet. This hypothesis appears to be favored by the circumstance that on the negatives made by ultra-violet light (and to a less degree by violet) the sky between the ball of the planet and the ring is distinctly denser than the region just outside. This would indicate that the region inside of the ring was filled with some material which reflected the short wave-lengths to a slight degree. No trace of this darkening appears on any of the plates made with the yellow screen, even on one that was many times overexposed, which

appears to show that the phenomenon is real. This peculiarity was, however, not detected until the work was finished, and I should prefer to verify it, or have it verified, before setting it down as an established fact. The luminosity is much too feeble to show in the prints. Dr. Hale will have the density measured with the Hartman photometer as soon as some repairs are finished. I do not, however, believe that the dark belt is in reality due to the absorption of a dust ring, for measurements made by Mr. Ellerman showed that the belt extended higher up on the ball of the planet than the line of intersection of the plane of the rings. Moreover it seems highly probable that the belt and the dark polar cap are to be explained in the same way. The second hypothesis assumes the existence in the planet's atmosphere of some substance capable of absorbing violet and ultra-violet light. This material might be a fine mist or dust, or some gas capable of absorbing the more refrangible part of the spectrum. Such a gas would be of a pale yellow color, and sulphur vapor and chlorine naturally occur to us.

I have, however examined the absorption spectrum of both of these gases in quartz bulbs, with the result that the absorption appears to be much stronger in the ultra-violet than in the violet, which is in disagreement with the circumstance that the band appears wider in violet than in ultra-violet light. Until spectrograms are available showing the complete spectra of the different zones of the planet it is useless to speculate as to the nature of the material which causes the absorption. On the infra-red photograph the ball of the planet is much brighter in



MONOCHROMATIC PHOTOGRAPHS OF SATURN, OCTOBER 23, 1915

comparison to the brightest part of the ring than on the violet and ultra-violet pictures. This again suggests a mist or dust in the planet's atmosphere which scatters the shorter wave-lengths. My infra-red photographs of landscapes have clearly demonstrated that we can obtain clear photographs through a blue haze by means of the spectrum region above 7200. In these photographs it will be remembered that the blue sky comes out black and the grass and foliage snow white. (See my *Physical Optics*, page 626, second edition.)

Interesting results were obtained as well in the case of Jupiter. In infra-red light the belts were scarcely visible, while the violet and especially the ultra-violet pictures showed dark belts, of which no trace could be seen in pictures made with the yellow screen, or by eye observations of the planet.

In view of the interesting results obtained with these two planets I hope that similar observations will be made of Mars on the occasion of its next near approach to the earth. I made one photograph of the planet in October with ultra-violet light, but the disc was too small to show much of interest.

I have been aided in this work by a grant from the Gould Fund of the National Academy of Sciences.